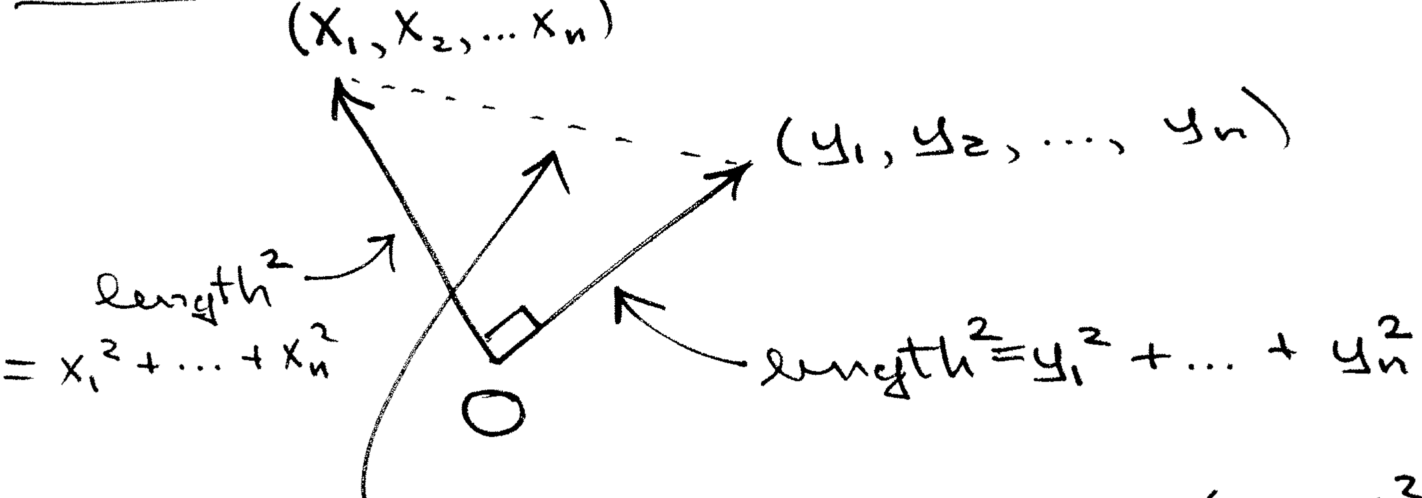
# Orthogonality and sums of squares

Two vectors $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ are orthogonal if $\sum_{i=1}^{n} x_i y_i = 0$

<u>Here's why</u>.




length$^2$ $(x_1 - y_1)^2 + \ldots + (x_n - y_n^2)$

$$= x_1^2 + \ldots + x_n^2 + y_1^2 + \ldots + y_n^2$$

$$- 2x_1 y_1 - 2x_2 y_2 - \ldots - 2x_n y_n$$

Pythagorean theorem: If the angle at O is $90^\circ$, then the square of the lengths of the two legs = square of length of hypotenuse

Therefore $-2\sum x_i y_i = 0$

Model 1 $\quad z_i = \mu + \alpha x_i + \varepsilon_i$

Model 2 $\quad z_i = \mu + \beta y_i + \varepsilon_i$

Model 3 $\quad z_i = \mu + \alpha x_i + \beta y_i + \varepsilon_i$

Model 3 SS: $\quad \sum (z_i - \mu - \alpha x_i - \beta y_i)^2$

$$= z_i^2 + \mu^2 + \alpha^2 x_i^2 + \beta^2 y_i^2$$
$$- 2 z_i \mu - 2\alpha z_i x_i - 2\beta z_i y_i$$
$$+ 2\mu\alpha x_i + 2\mu\beta y_i$$
$$+ 2\alpha\beta x_i y_i$$

If $(x_i)$ and $(y_i)$ are orthogonal

then $\quad 2\alpha\beta \sum x_i y_i = 0$

Result: ~~terms with~~ the SS is separable into terms involving $\alpha$ and terms involving $\beta$ —

(Model 3) SS = (Model 1) SS + (Model 2) SS

for ANY $\alpha$ $\beta$.