Homework Assignment 3 (Due October 8)

(a) All HW problems in Lecture 4

(b) Let $U$ be an rv with a Uniform(0,1) distribution. For each integer $n \geq 1$, define
$$X_n = \left(n/\log(n)\right)I_{[0, \frac{1}{n}]}(U) + 1.$$

(a) Does $X_n \xrightarrow{d} X$? (if so, identify the distribution of $X$)
(b) Does $X_n \xrightarrow{p} X$? (If so, identify $X$)
(c) Compute $E(X_n)$. Does it converge to $E(X)$ for some $X$?

(c) Suppose that, conditional on probabilities $p_1, p_2, \ldots, p_n$, the binary rvs $X_1, X_2, \ldots, X_n$ are independent with $\Pr(X_i = 1|p_1, p_2, \ldots, p_n) = p_i$, $\Pr(X_i = 0|p_1, p_2, \ldots, p_n) = 1 - p_i$, $i = 1, 2, \ldots, n$.

Now, suppose that each $p_i$ is a random draw from a distribution with mean $p$. What is the marginal distribution of $S_n = X_1 + X_2 + \ldots + X_n$?