This is a open-book, open-notes, computer-assisted examination. Please write up your results in report form using Word or LaTeX, and turn in a hard copy to the proctor or email a .pdf or .doc file to quic0038@umn.edu by 3:55 pm today. That is, you will have 85 minutes (2:30–3:55 pm) to complete this data analysis and write up its results. Good luck.

<table>
<thead>
<tr>
<th>week ($X_j$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>game 1 ($Y_{1j}$)</td>
<td>85</td>
<td>78</td>
<td>64</td>
<td>94</td>
<td>87</td>
<td>110</td>
<td>122</td>
<td>100</td>
<td>110</td>
<td>123</td>
<td>105</td>
<td>112</td>
<td>103</td>
<td>140</td>
</tr>
<tr>
<td>game 2 ($Y_{2j}$)</td>
<td>136</td>
<td>142</td>
<td>87</td>
<td>129</td>
<td>113</td>
<td>65</td>
<td>110</td>
<td>103</td>
<td>92</td>
<td>133</td>
<td>118</td>
<td>94</td>
<td>132</td>
<td>125</td>
</tr>
<tr>
<td>game 3 ($Y_{3j}$)</td>
<td>92</td>
<td>127</td>
<td>136</td>
<td>149</td>
<td>119</td>
<td>148</td>
<td>120</td>
<td>159</td>
<td>127</td>
<td>121</td>
<td>117</td>
<td>153</td>
<td>118</td>
<td>108</td>
</tr>
</tbody>
</table>

1. Katie bowled $G = 3$ games each week for $W = 14$ weeks; the data are given in the table above and online at www.biostat.umn.edu/~brad/data/bowling_data.txt. Shown are the week, $X_j$, and the three weekly scores, $Y_{ij}$, for $i = 1, 2, 3$, and $j = 1, \ldots, 14$.

Having previously analyzed the score data itself over time, Katie now wishes to understand the temporal pattern of improvements in her scores from Game 1 to Game 2, and also from Game 2 to Game 3. To do this, she defines two sets of binary variables,

$$Z_{1j} = \begin{cases} 1 & \text{if } Y_{2j} - Y_{1j} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$Z_{2j} = \begin{cases} 1 & \text{if } Y_{3j} - Y_{2j} > 0 \\ 0 & \text{otherwise} \end{cases}.$$

She then hopes to model $p_{ij} = P(Z_{ij} = 1)$ as two independent logistic regressions,

$$\logit(p_{ij}) = \log[p_{ij}/(1 - p_{ij})] = \beta_{0i} + \beta_{1i}(X_j - \bar{X}), \quad \text{for } i = 1, 2 \quad \text{and } \quad j = 1, \ldots, 14,$$

where $X_j = j$ (the week index), which we center around its own mean to ease collinearity between $\beta_{0i}$ and $\beta_{1i}$.

(a) Use R to make a plot of $X$ versus $Z_1$, and a second plot of $X$ versus $Z_2$. Use the `glm` command with the `family=binomial("logit")` option to fit the two logistic regressions above using classical methods. Then use `lines` command to add the fitted logistic regression lines (available using the `fitted.values` output option in `glm`) to your plots. Are there (classically) significant time trends in either of the two score improvement indicators?

(b) Fit the two logistic regressions in OpenBUGS, using vague priors for all parameters. Use 3 parallel MCMC chains each with their own distinct initial values, and justify the convergence of your algorithm. Find and compare the posterior density estimates and 95% credible intervals for week effect parameters $\beta_{1i}, i = 1, 2$. Again comment on whether there are significant time trends in score improvement, and any apparent differences between your new Bayesian answers to this question and the classical ones in part (a).

(c) Use your posterior means for the $\beta_{0i}$ and $\beta_{1i}$ to compute Bayesian fitted values $\hat{p}_{ij}$ for all $i$ and $j$, and then add these fitted values to your plots in (a) using the `points` command. Do these Bayesian fitted values agree with your classical ones?
(d) Finally, obtain point estimates of

\[ q_i = P(\beta_{1i} < 0 \mid Z_{i1}, \ldots, Z_{i14}) \]

for \( i = 1, 2 \). Describe any differences across \( i \), and interpret your findings.

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**Overall Hint:** Nice models for the R and **BUGS** code you will need for this problem may be found in the two “binary dugongs” examples,

- www.biostat.umn.edu/~brad/data/dugongsBin_R.txt,

and

- www.biostat.umn.edu/~brad/data/dugongsBin_BUGS.txt,

respectively.