**Instructions:** This is a take-home exam. Please write up your answers clearly in report form using Word or LATEX, and turn in a hard copy to me or email a .pdf or .doc file to quic0038@umn.edu before 11:55 pm April 10th, 2012. **No extensions will be provided.** It is required that you work independently on this exam: do not consult among yourselves and also do not consult the TA, Mr. Harrison Quick. Any actions contradicting this spirit will constitute a violation of the student honor code. Results without commentary and interpretation will not receive full credit. If you need clarifications, contact Professor Carlin (brad@biostat.umn.edu). Good luck!

1. Consider again the model of Example 7.2, illustrated by the famous “rat data.” Suppose that instead of a straight-line growth curve, we wish to investigate the exponential growth model,

\[ Y_{ij} \overset{\text{ind}}{\sim} N \left( \alpha_i e^{\beta_i (x_{ij} - \bar{x})}, \sigma^2 \right), \quad i = 1, \ldots, k; \quad j = 1, \ldots, n_i, \]

with the remaining priors and hyperpriors as previously specified.

(a) Of the full conditional distributions for \( \{\alpha_i\}, \{\beta_i\}, \alpha_0, \beta_0, \Sigma^{-1} \) and \( \sigma^2 \), which now require modification?

(b) Of those distributions in your previous answer, which may still be derived in closed form as members of familiar families? Obtain expressions for these distributions.

(c) Of those full conditionals that lack a closed form, give a Metropolis or Hastings substep for generating the necessary MCMC samples.

(d) Write a **BUGS** or **R** program applying your results to the rat data, as given in Table 7.3, on the web at [http://www.biostat.umn.edu/~brad/data/rat_data.txt](http://www.biostat.umn.edu/~brad/data/rat_data.txt), or in the **rats** example in **BUGS Examples Volume 1**. Obtain estimated posterior distributions for \( \alpha_0 \) and \( \beta_0 \), and compare model fit and predictive ability to that of the linear model considered in Example 7.2. What do your results suggest about the growth patterns of young rats?

2. Consider again Fisher’s sleep data:

\[
1.2, 2.4, 1.3, 1.3, 0.0, 1.0, 1.8, 0.8, 4.6, 1.4 .
\]

Suppose these \( k = 10 \) observations arose from the Gaussian/Gaussian PEB model,

\[
Y_i | \theta_i \overset{\text{ind}}{\sim} N(\theta_i, \sigma^2), \quad i = 1, \ldots, k ,
\]

\[
\theta_i \overset{iid}{\sim} N(\mu, \tau^2), \quad i = 1, \ldots, k ,
\]

where we will assume \( \sigma^2 = 1 \).

(a) For these data and model, compute the two classic PEB point estimators \( \hat{\theta}^{JS}(Y) \), given on p.237 of the CL3 text, and \( \hat{\theta}^{JS'}(Y) \), given on p.238. Compare your answers across \( i \); do they behave as expected?
(b) Write an R program to simulate the frequentist risk of the two estimators,

\[
R(\theta, \hat{\theta}(Y)) = \frac{1}{k} \sum_{i=1}^{k} E(\hat{\theta}_i(Y) - \theta_i)^2 | \theta)
\]

where we assume \( \theta_1 = \cdots \theta_{10} = 0 \). Which estimator performs better and why?

(c) Repeat the experiment of part (b), but now for \( \theta_1 = \cdots \theta_9 = 0 \) and \( \theta_{10} = 10 \). Which estimator performs better now, and why?

(d) Repeat the experiment in part (c), but now consider the *component-specific* frequentist risk,

\[
R(\theta_i, \hat{\theta}_i(Y)) = E((\hat{\theta}_i(Y) - \theta_i)^2 | \theta)
\]

for \( i = 1 \) and \( i = 10 \). Which estimator performs better in each case, and why?

(e) Repeat the experiment of part (b), but now for *EB (preposterior)* risk,

\[
r(\hat{\theta}(Y)) = E_{\theta} \left[ R(\theta, \hat{\theta}(Y)) \right] = \frac{1}{k} \sum_{i=1}^{k} E_{\theta}[E((\hat{\theta}_i(Y) - \theta_i)^2 | \theta)]
\]

as indicated on p.260. Assume \( \mu = 2 \) and \( \tau^2 = 1 \). Which estimator performs better now, and why?