STAT 8311/PUBH 8401 HW 10

Due Nov. 30, 2022

Exercise 1. Show that if $\mathcal{E}$ is a linear subspace of $\mathbb{R}^n$ where $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$ for 2 linear subspaces $\mathcal{E}_1$ and $\mathcal{E}_2$, then if P_X is the usual projection onto X, $P_{\mathcal{E}_1} P_{\mathcal{E}} = P_{\mathcal{E}_1}$.

Exercise 2. Using the result from the previous exercise show that $Q_{\mathcal{E}_1} Q_{\mathcal{E}} = Q_{\mathcal{E}}$, where Q is the usual orthogonal projection.

Exercise 3. Suppose we have a model where the vector y depends on a p-level factor and a collection of continuous predictor variables Z_i for $i = 1, \ldots, m$. So we can write

$$y = X\beta + Z\gamma + \epsilon.$$

Let $\mathcal{E}_1$ be the space spanned by the columns of X and $\mathcal{E}_2$ the space spanned by the columns of Z and suppose both β and γ are estimable. If $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$ then show that

$$Q_{\mathcal{E}} y = Q_{\mathcal{E}_1} y - Q_{\mathcal{E}_1} Z\hat{\gamma}.$$

(Hint: use the result from the previous problem.)

Exercise 4. Continuing with the previous problem, show that

$$Z^T (Q_{\mathcal{E}_1} y - Q_{\mathcal{E}_1} Z\hat{\gamma}) = 0.$$

Exercise 5. From the previous exercise, show that if $Z^T Q_{\mathcal{E}_1} Z$ is full rank then $\hat{\gamma} = (Z^T Q_{\mathcal{E}_1} Z)^{-1} Z^T Q_{\mathcal{E}_1} y$.

Exercise 6. Using the result from the previous exercise, find an explicit expression for $\hat{\gamma}$ if there is a single z variable and the experiment is balanced for the factor encoded by X.

(Hint: using notation like (y_{ij}, z_{ij}) for group i and replicate j for the ANOVA part of the design makes this a little simpler.)

Exercise 7. Show that $Z^T Q_{\mathcal{E}_1} Z$ has the same rank as $Q_{\mathcal{E}_1} Z$. Show that the latter matrix is full rank if Z is full rank.