STAT 8311/PUBH 8401 HW 9

Due Nov. 23, 2022

Exercise 1. Let x_1 and x_2 be 2 vectors in $\mathbb{R}^2$ and let θ be the angle between them, measured counter-clockwise from x_1 to x_2. Show that $\cos(\theta) = \frac{(x_1, x_2)}{||x_1|| ||x_2||}$ and deduce that x_1 and x_2 are orthogonal if and only if $\theta = 90°$ or $\theta = 270°$. (Hint: $\cos(\theta_2 - \theta_1) = \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)$.)

Exercise 2. Suppose we have the model $y_{ij} = \mu_0 + \beta_i + \epsilon_{ij}$ for $i = 1, \ldots, p$ and $j = 1, \ldots, n$ where ϵ_{ij} are independently distributed according to a 0 mean normal distribution with variance σ^2. We've seen that the OLS estimate of $\mathrm{E}y_{ij}$ is given by $\frac{1}{n}\sum_j y_{ij}$. Why can't one obtain unique estimates of μ_0 and β_i for all i without additional constraints? If we assume $\beta_1 = 0$ provide unbiased estimates of μ_0 and β_i for all i. If we assume $\sum_i \beta_i = 0$ provide unbiased estimates of μ_0 and β_i for all i.

Exercise 3. Show that the Kronecker product of 2 idempotent matrices is also idempotent.

Exercise 4. Show that the Kronecker product of 2 orthogonal matrices is also orthogonal.

Exercise 5. Let A and B be 2 matrices. Show that the singular value decomposition of $A \otimes B$ can be expressed in terms of the elements of the singular value decomposition of A and B, and use this to express the eigenvalues of $A \otimes B$ in terms of the eigenvalues of A and B.

Exercise 6. Consider the regression model $y_i = \beta x_i + \epsilon_i$ where ϵ_i are iid zero mean errors with variance σ^2. Find an expression for the F test of the null hypothesis that $\beta = 0$.

Exercise 7. On page 115 of the course notes a reparameterization trick is introduced as a way to simplify a test of a null hypothesis that takes the form $A_1\beta = 0$. The course notes state that if we construct A_0 so that $A_0^T A_1 = 0$ then $Z_0^T Z_1 = 0$ and this leads to a simplification in the F test. Is the statement that $A_0^T A_1 = 0$ implies $Z_0^T Z_1 = 0$ true? If so prove it, otherwise provide a counterexample. If it is not true, is there a sufficient condition that makes this true?