PubH 7405: REGRESSION ANALYSIS

SLR: DIAGNOSTICS & REMEDIES

Normal Error Regression Model:

$$\mathbf{Y} = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \mathbf{x} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} \in \mathbf{N}(\mathbf{0}, \boldsymbol{\sigma}^2)$$

The Model has <u>several parts</u>: Normal Distribution, Linear Mean, Constant Variance, etc…

In doing statistical analyses, a “statistical model” – e.g. “normal error regression model”- is absolutely necessary.

However, a model is just an assumption or a set of assumptions about the population of which data we have are considered as a sample; they may or may not fit the observed data. Certain part or parts of a model may be violated and, as a consequence, results may not be valid.

IMPORTANT QUESTIONS

Does the Regression Model fit the data?

Then what if the Regression Model, or certain part of the Regression Model, does not fit the data ? i.e. (1) If it does not fit, could we do something to make it fit? And (2) Does it matter if it still does not fit?

POSSIBLE DEPARTURES FROM THE NORMAL REGRESSION MODEL

- The regression function is not linear
- Variance (of error terms) is not constant
- Model fits all but a few "outliers"
- Responses are not independent
- Responses are not normally distributed

Outliers and missing predictor or predictors are not model's violation but might even have more severe consequences.

Besides the data values for the dependent and independent variables, diagnostics would be based on the “residuals” (errors of individual fitted values) and some of their transformed values.

SEMI-STUDENTIZED RESIDUALS

$\varepsilon \in N(0,\sigma^2)$

$\{e_i\}$ is a sample with mean zero →

$$\boxed{e_i^* = \frac{e_i - \bar{e}}{\sqrt{MSE}} = \frac{e_i}{\sqrt{MSE}}}$$

If $\sqrt{MSE}$ were an estimate of the standard deviation of the residual e, we would call e* a studentized (or standardized) residual. However, **standard deviation of the residual is complicated and varies for different residuals**, and $\sqrt{MSE}$ is only an approximation. Therefore, e* is call a **“semi-studentized residual”.**

Diagnostics could be informal using plots/graphs or could be based on formal application of statistical tests; graphical method is more popular and would be sufficient. We could perform a few statistical tests but, most of the times, they are not really necessary.

PLOTS OF RESIDUALS

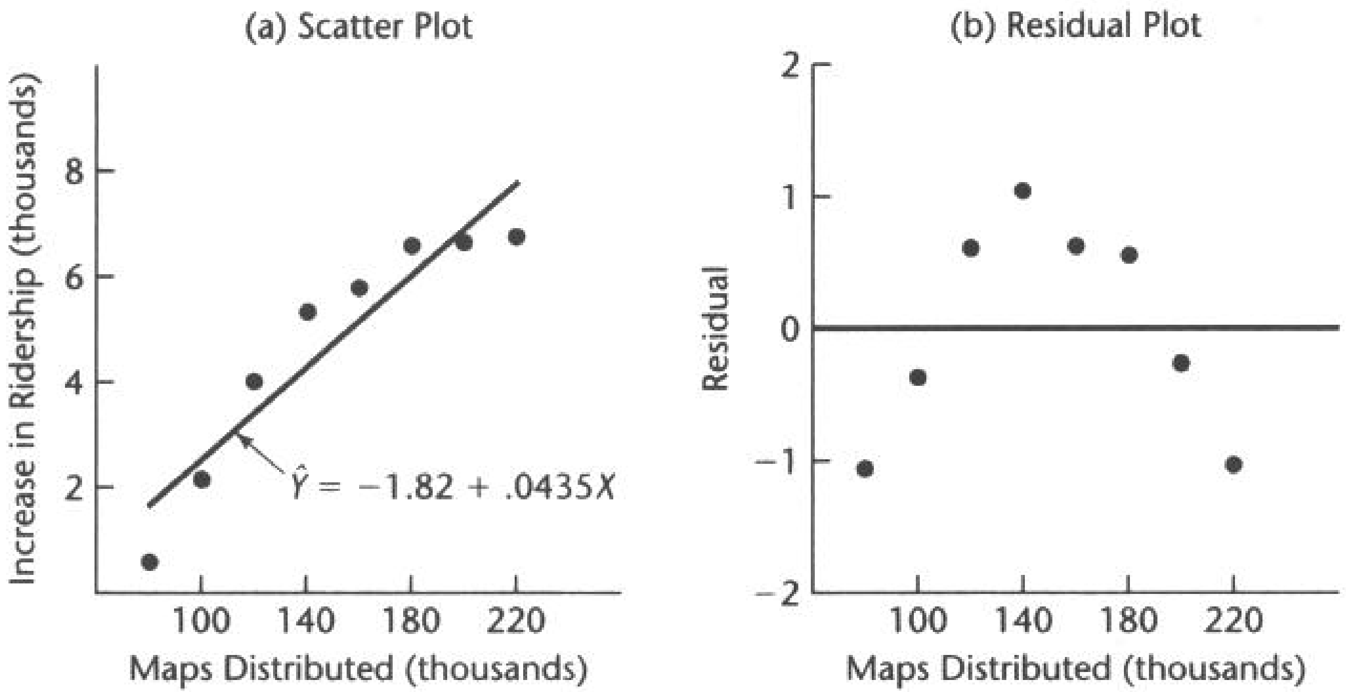
- Plot of residuals against predictor
- Plot of absolute/squared residuals against predictor
- Plot of residuals against fitted values
- Plot of residuals against time or other sequence.
- Plot of residuals against omitted predictor variable
- Box plot of residuals
- Normality plot of residuals

In any of those graphs, you could plot semi-studentized residuals instead of residuals. A semi-studentized residual is a residual on “standard deviation scale”; graphs provide same type of information.

Issue: NONLINEARITY

- Whether a linear regression function is appropriate for a given data set can be studied from a scatter diagram (e.g.. Using Excel); but it's not always effective (**less visible**).
- More effective to use a residual plot against the predictor variable or, equivalently, against the fitted values; **if model fits, one would have a horizontal band centered around zero which has no special clustering pattern.**
- **The lack of fit** would result in a graph showing the residuals departing from zeros in a systematic fashion – **likely a curvilinear shape.**

FIGURE 3.3 Scatter Plot and Residual Plot Illustrating Nonlinear Regression Function—Transit Example.



Easier to see; WHY?

REMEDIAL MEASURES

- If a SLR model is found not appropriate for the data at hand, there are **two basic choices**:

(1) **Abandon** it and search for a suitable one, or

(2) Use **some transformation** on the data to create a fit for the transformed data

- **Each has advantages & disadvantages**: first approach may yield better insights but may lead to more technical difficulties; transformations are more simple but may obscure the fundamental real relationship; sometimes **it's hard to explain**.

LOG TRANSFORMATIONS

- Typical: Y* = Log (Y), turns a multiplicative model into an additive model – **for linearity**.
- Residuals should be used to check if model fits transformed data: normality, independence, and constant variance because the distribution changes the distribution and the variance of the error terms.
- **Others:** (1) **X* = Log (X),**
  (2) X* = Log (X) and Y* = Log (Y);
  Example: Model (2) is used to study "demand" (Y) versus "price of commodity" (X) in economics.

Example:

$Y^* = \ln(Y=PSA)$ is used in the model for PSA with Prostate Cancer

Note: When the distribution of the error terms is close to normal with an approximately constant variance, and a transformation is needed only for linearizing a non-linear regression relation, only transformations on X should be attempted.

RECIPROCAL TRANSFORMATIONS

- Also aimed for linearity
- Possibilities are:

(1) $X^* = 1/X$,

(2) $Y^* = 1/Y$,

(3) $X^* = 1/X$ and $Y^* = 1/Y$

- **Example: Models (1) and (2) are useful when it seems that Y has a lower or upper "asymptote" (e.g. hourly earning)**

Logarithmic and Reciprocal Transformations can be employed together to linearize a regression function. For example, the “Logistic Regression Model” (with Y = probability/proportion “p”):

$$Y = \ln\left(\frac{p}{1-p}\right)$$

$$= \frac{1}{1+\exp(-\beta_0 - \beta_1 x)}$$

Issue: NONCONSTANCY OF VARIANCE

- **Scatter diagram** is also helpful to see if the variance of error terms are constant; **if model fits**, one would have a **band with constant width centered around the regression line** which has **no** special clustering pattern. Again, not always effective
- More effective to **plot residuals (or their absolute or squared values) against the predictor variable** or, equivalently, against the fitted values. **If model fits**, one would have a **band with constant width centered around the horizontal axis. The lack of fit would result in a graph showing the residuals departing from zeros in a systematic fashion – likely a "megaphone" or "reverse megaphone" shape.**

EXAMPLE: Plutonium Measurement

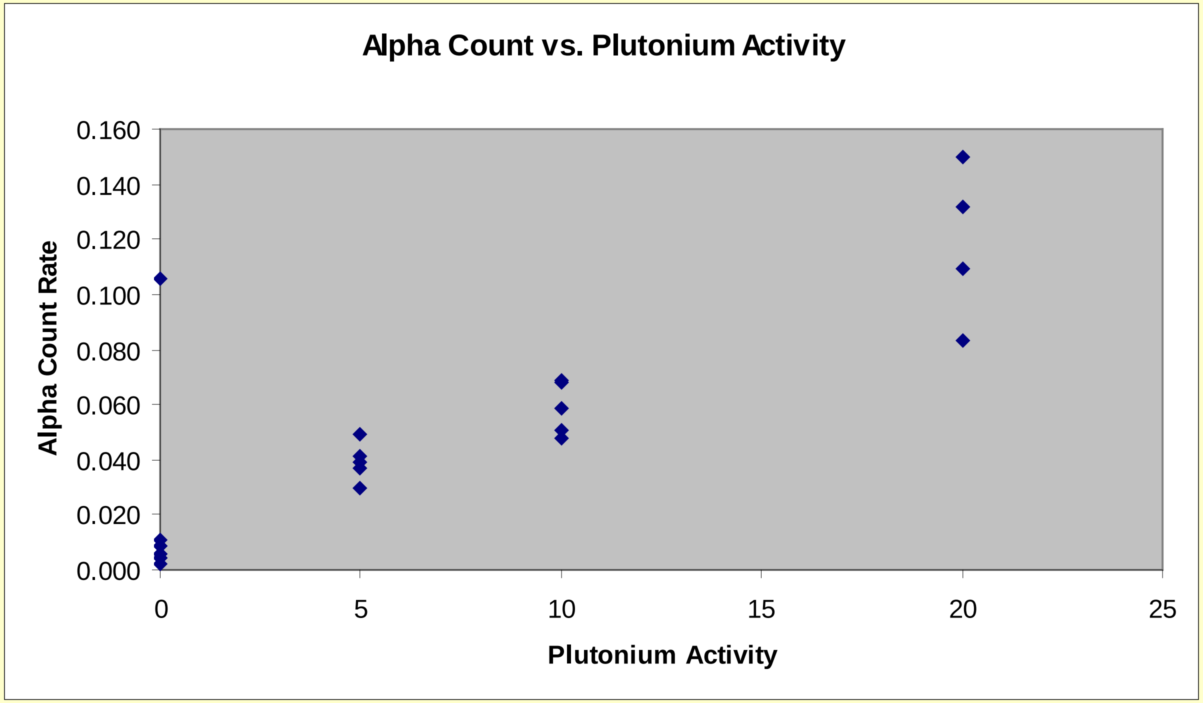
An Example in environmental clean up;

X = Plutonium Activity (pCi/g)

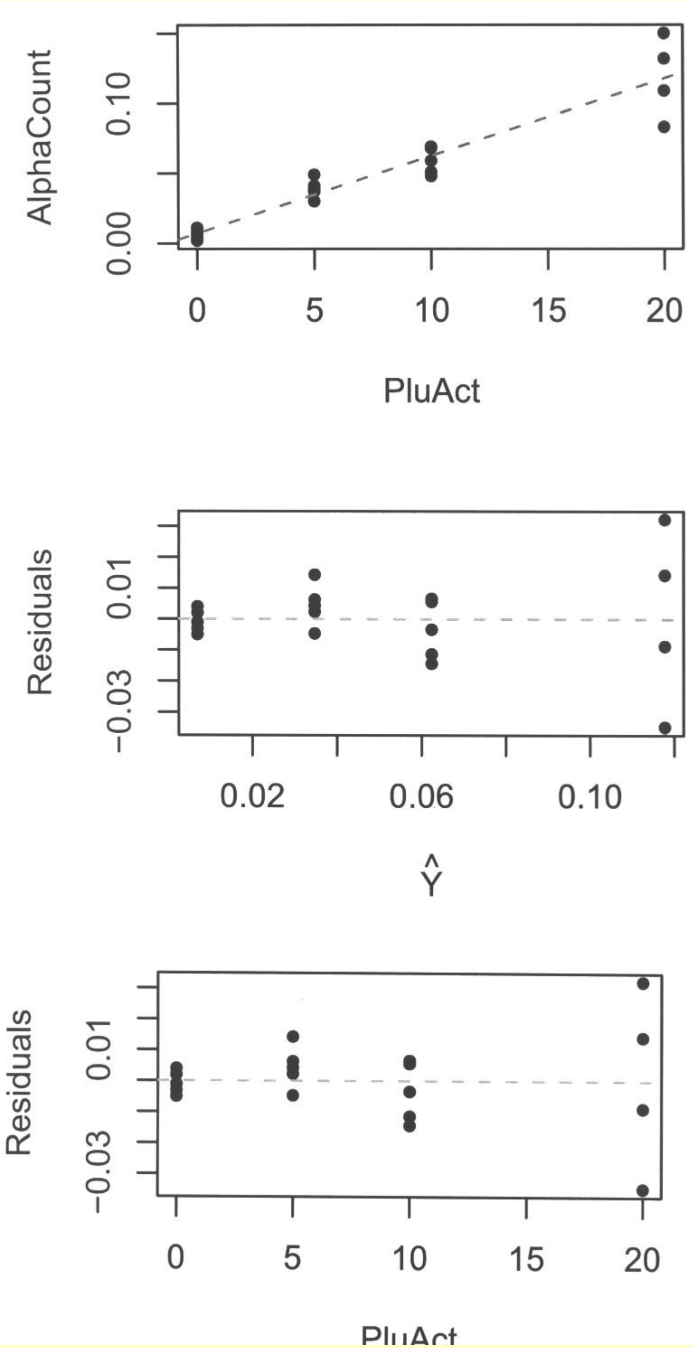
Y = Alpha Count Rate (#/sec)

A full description of the example is in section 3.11, starting on page 141 (in practice its use involves an inverse prediction, predicting plutonium activity from the observed alpha count (Plutonium emits alpha particles).

Plutonium (X) Activity (pCi/g)	Alpha Count Rate (#/sec)
20	0.150
0	0.004
10	0.069
5	0.030
0	0.011
0	0.004
5	0.041
20	0.109
10	0.068
0	0.009
0	0.009
10	0.048
0	0.006
20	0.083
5	0.037
5	0.039
20	0.132
0	0.004
0	0.006
10	0.059
10	0.051
0	0.002



Scatter Diagram



Residual Plots

Easier to see -
Same reason

TESTS FOR CONSTANT VARIANCE

- If variance is not constant, coverage of confidence intervals might be affected.
- There are many tests for non-constant variance but two are often mentioned
- The Breusch-Pagan test **assumes normality of error terms** but the **test follows the usual regression methodology – not hard to do**.
- The Brown-Forsythe test **does not depend on normality of error terms**; this is desirable because non-constant variance and non-normality tend to go together. This test is easy.

BROWN-FORSYTHE TEST

- The Brown-Forsythe test is used to ascertain whether the error terms have constant variance; especially when the **variance of the error terms either increases or decreases with the independent variable X.**
- The Test: divide the data into 2 groups, say **half with larger values of X** and **half with smaller values of X**; **(1) calculating the “absolute deviations” of the residuals around their group mean (or median); (2) applying the two-sample t-test.**
- Test statistic follows approximately the t-distribution when the variance of the error terms is constant (under the Null Hypothesis) and the sizes of the two group are not extremely small.

BROWN-FORSYTHE: RATIONALE

- If the error variance is either increasing or decreasing with X, the residuals in one group tend to be more variable than those residuals in the other.
- The Brown-Forsythe test does not assume normality of error terms; this is desirable because non-constant variance and non-normality tend to go together.
- It's is very similar to **"Levine's test"** to **compare any two variances – instead of forming the ratio of two sample variances (& use "F-test").**

EXAMPLE: **Toluca Company Data** (Description on page 19 of Text)

LotSize	WorkHours
80	399
30	121
50	221
90	376
70	361
60	224
120	546
80	352
100	353
50	157
40	160
70	252
90	389
20	113
110	435
100	420
30	212
50	268
90	377
110	421
30	273
90	468
40	244
80	342
70	323

Group 1: n = 13 with lot sizes from 20 to 70; median residual = -19.88

Group 2: n = 12 with lot sizes from 80 10 120; median residual = -2.68

Mean of absolute residuals :

$$\text{Group 1: } 44.815$$

$$\text{Group 2: } 28.450$$

$$\text{Pooled Variance : } 964.21; \; s_p = 31.05$$

$$t = \frac{44.815 - 28.450}{31.05\sqrt{\frac{1}{13} + \frac{1}{12}}}$$

$$= 1.32$$

$$\text{two-sided p-value} = .20$$

This example shows that **the half with smaller X’s has larger residuals** – and vice versa; **the pattern of an inverse mega phone** – but it’s “**not significant**”, a case that makes me uneasy with statistical tests: I want to assume that the variance is constant, **it only says that we do not have enough data to conclude that the variance is not constant!**

WEIGHTED LEAST SQUARES

- Constant variance = Homoscedasticity
- Non-constant variance = Heteroscedasticity
- Most often: Variance is functionally related to the mean; e.g. standard deviation or variance is proportional to X. A possible solution is performing “weighted” least-squares estimation instead of “ordinary”

With **ordinary least squares**, estimators for regression coefficients are obtained by minimizing the quantity Q; setting the partial derivatives equal to zero to have the “normal equations”:

$$Q = \sum (Y - \beta_0 - \beta_1 X)^2$$

With **weighted least squares**, estimators for regression coefficients are obtained by minimizing the quantity Q where “w” is a “**weight**” (associated with the error term); setting the partial derivatives equal to zero to have the “normal equations”:

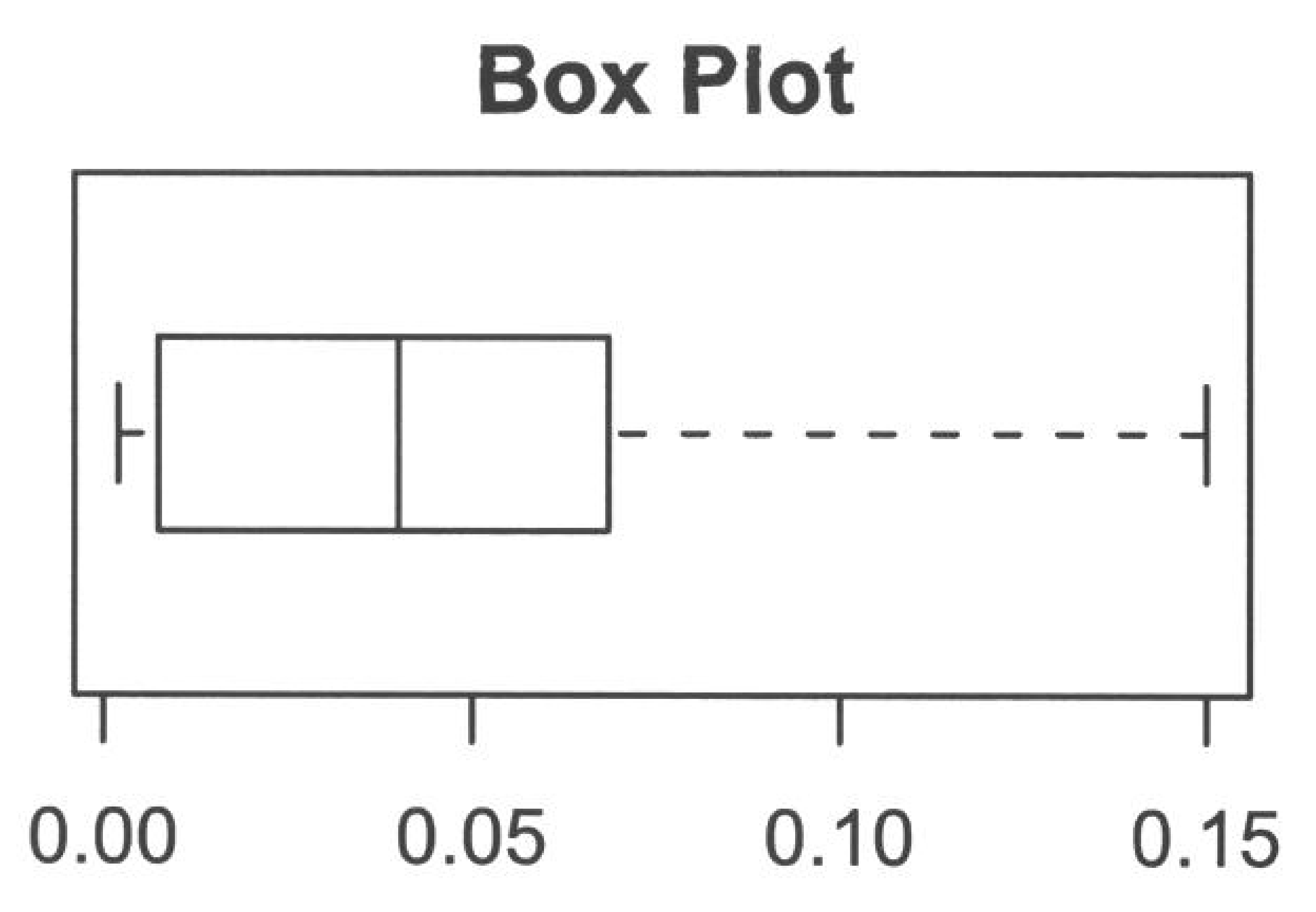
$$Q = \sum w(Y - \beta_0 - \beta_1 X)^2$$

The optimal choice for the weight is the inverse of the variance; when the variance is constant, ordinary and weighted least squares estimators are identical. For example, **when standard deviation is proportional to X** (variance is kX^2), we minimize:

$$Q = \sum \frac{1}{kX^2}(Y - \beta_0 - \beta_1 X)^2$$

ISSUE: PRESENCE OF OUTLIERS

- **Outliers are extreme observations**
- They can be identified from a Box plot or a residual plot graphing semi-studentized residuals against independent variable values or fitted values.
- Point with residuals representing 3-4 standard deviations from their fitted values are suspicious.
- **Presence of outliers could cause the impression that a linear regression model does not fit.**


Box Plot
0.00
0.05
0.10
0.15

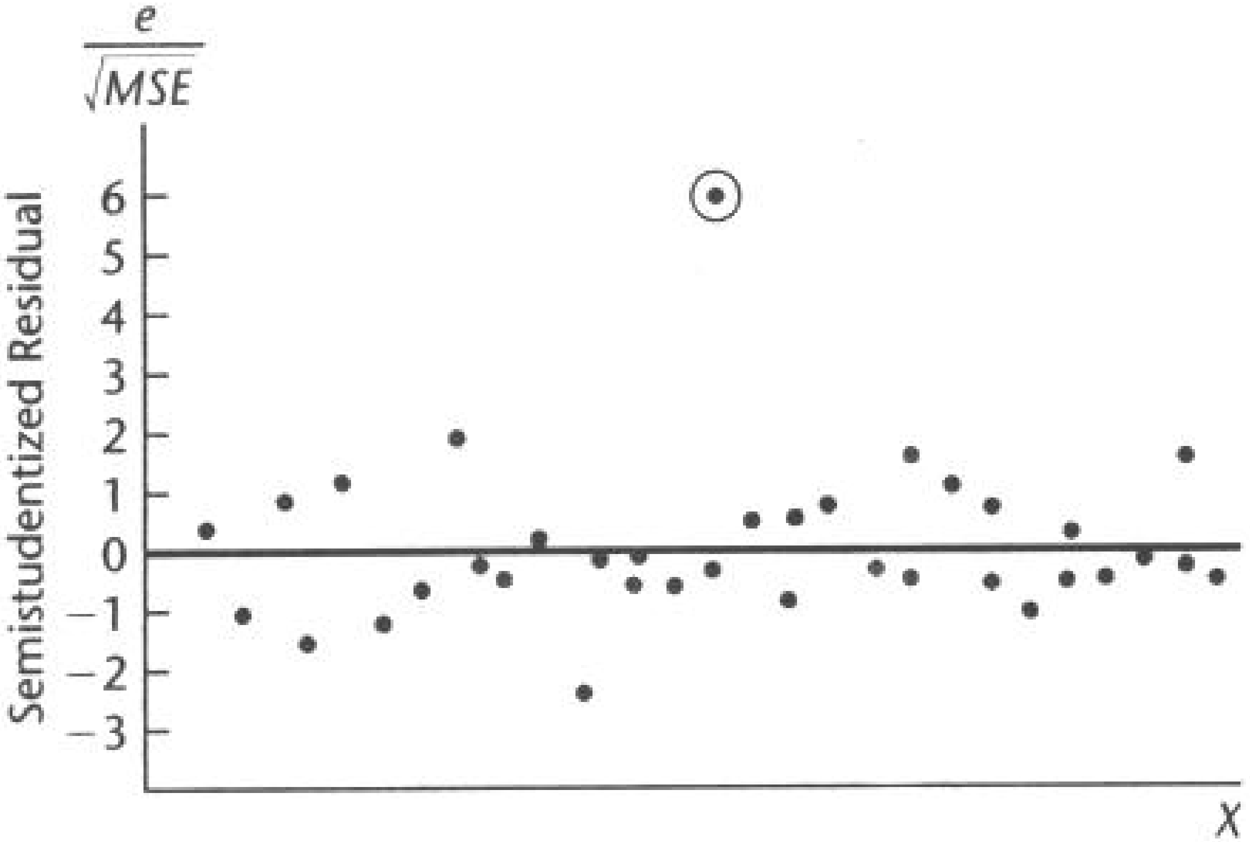
In a Box Plot:

(1) The box extends from first quartile to third quartile, divided into 2 parts at the median,

(2) Two lines (or the “whiskers”) projecting out from the box extending to both sides, each by a distance equal to 1.5 times the length of the adjacent compartment

(3) It tells about “symmetry” of the distribution – those points beyond the reach of the whiskers are usually considered **“extreme”**

FIGURE 3.6
Residual Plot with Outlier.



It is extremely hard to deal with outliers:

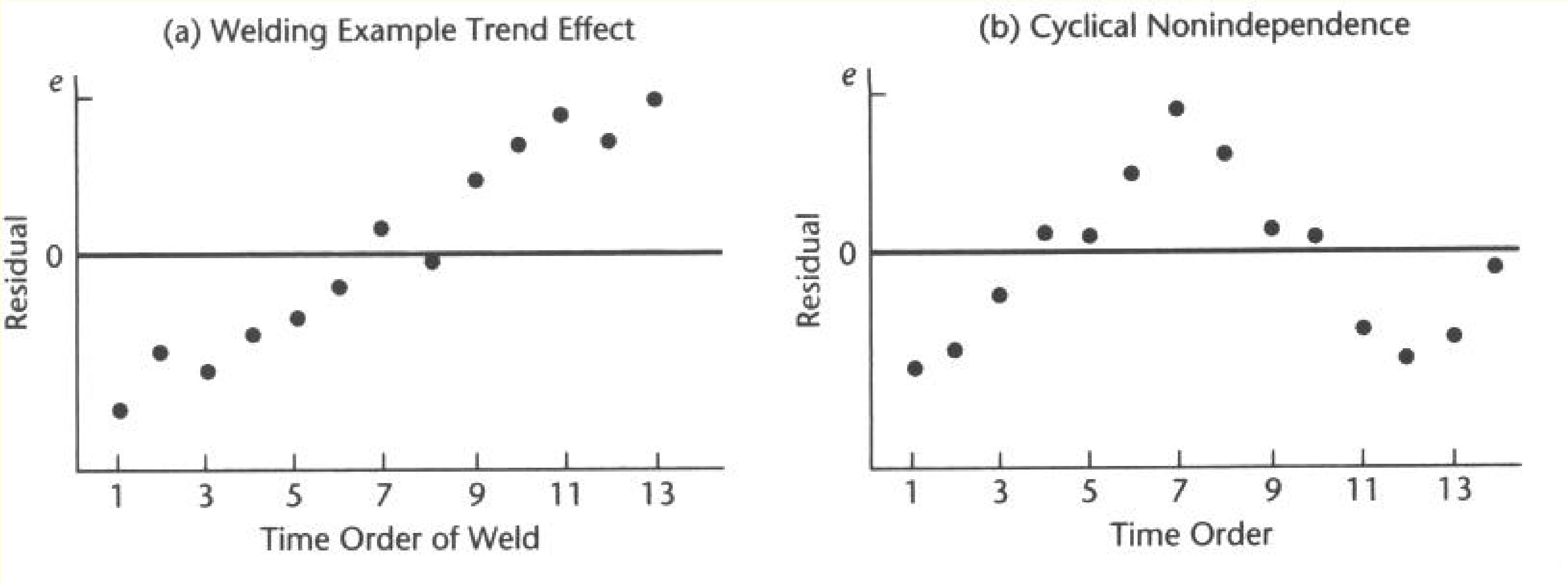
(1) Some are simple **results of mistakes or recording errors**; as such, they should be discarded.

(2) **Some may convey important information**: an outlier may occur because of an interaction with another independent variable not under investigation.

A safe rule is to discard an outlier only if there is direct evidence that it represents a error or miscalculation.

ISSUE: NONINDEPENDENCE OF ERROR TERMS

- Whenever data are obtained in a **time sequence** or some other type of sequence – such as **adjacent geographical areas**, it is a good idea to prepare a sequence plot of the residuals (**residuals vs. time**)
- When the error terms are independent, the residuals in such a graph fluctuate in a random pattern; lack of randomness shows in the form of a time trend or cyclical pattern .
- This is the special case of a predictor omitted from the regression model (in this case, it's "time").



(a) Welding Example Trend Effect

(b) Cyclical Nonindependence

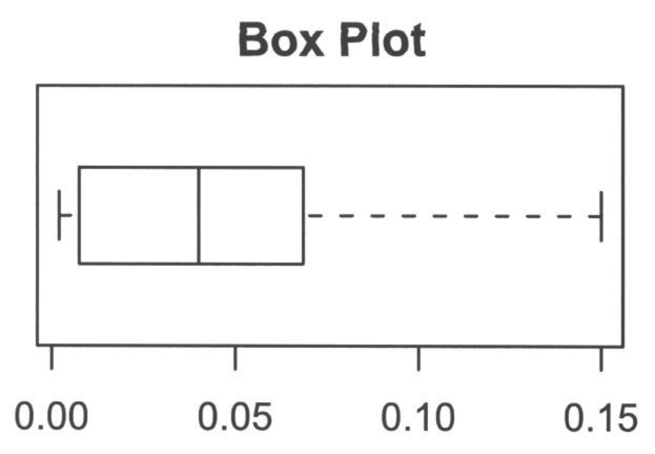
ISSUE: NONNORMALITY OF ERROR TERMS

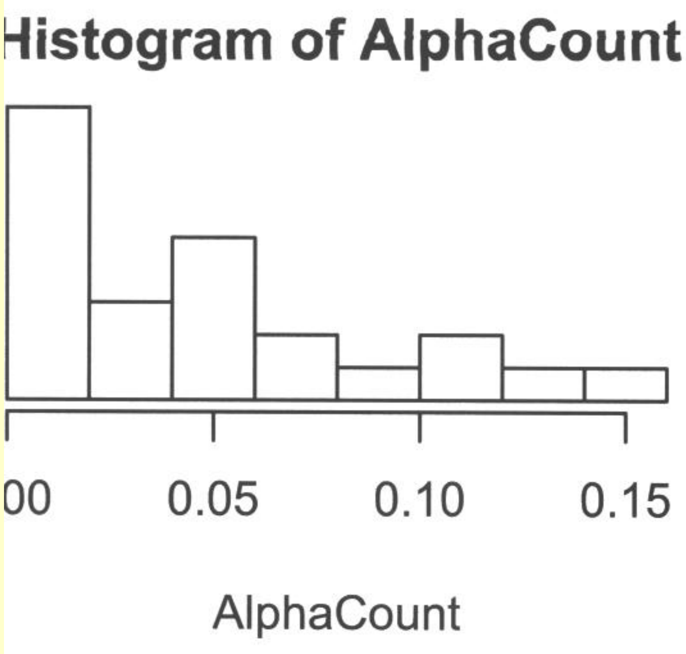
BASIC TOOLS:

(1)Histogram,

(2)Stem-and-Leaf Plot, &

(3)Box Plot





Stem-and-Leaf Plot

```
decimal point 1 digit to the right of |
0 | 0000111113444
0 | 5556778
1 | 113
1 | 5
```

In a Box Plot:

(1) The box extends from first quartile to third quartile, divided into 2 parts at the median,

(2) Two lines (or the “whiskers”) projecting out from the box extending to both sides, each by a distance equal to **1.5 times the length of the adjacent compartment**

(3) It tells about “**symmetry**” of the distribution – those **points beyond the reach of the whiskers are usually considered “extreme”**

Issue: DEPARTURE FROM NORMALITY

Violation of the normality assumption can be checked more effectively using the normal probability plot. **Each residual is plotted against its expected value under normality** (the “Normal Q-Q Plot”). A plot that is nearly **linear** suggests agreement with the normality assumption, whereas **a plot that departs substantially from linearity suggests that the distribution is not normal.**

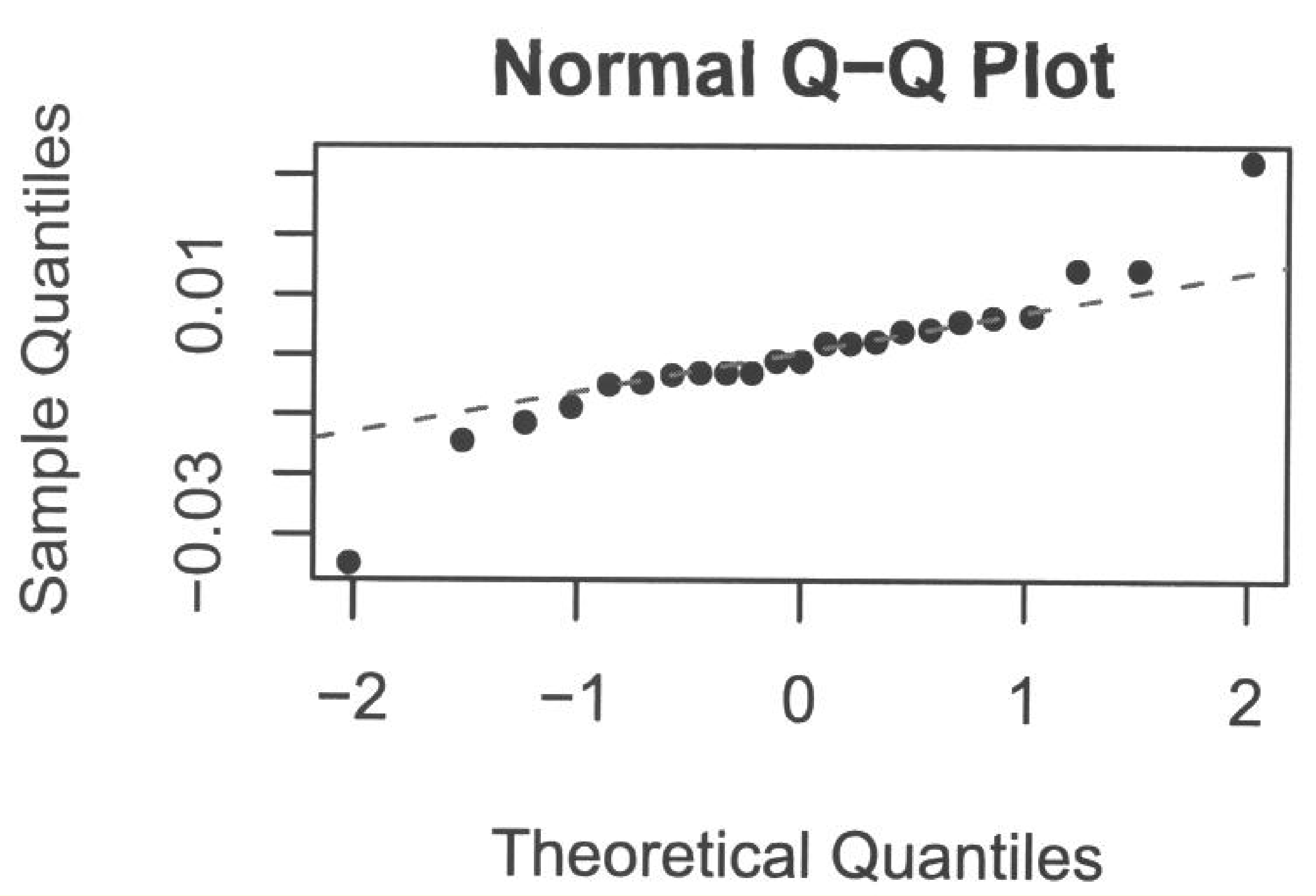
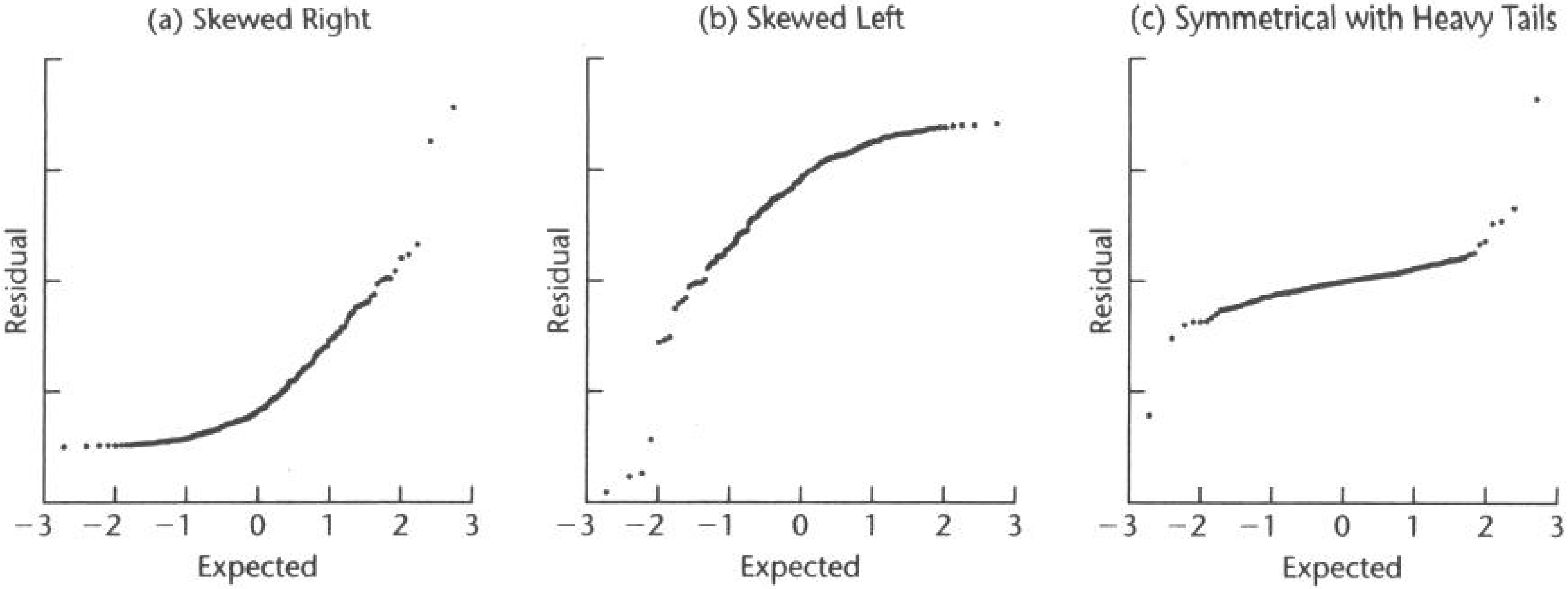

Normal Q−Q Plot
Sample Quantiles
−0.03
0.01
−2
−1
0
1
2
Theoretical Quantiles

FIGURE 3.9 Normal Probability Plots when Error Term Distribution Is Not Normal.



TESTS FOR NORMALITY

- Goodness-of-fit tests – such as the Kolmogorov-Smirnov test – can be used for examining the normality of the error terms; but they are a bit advanced for first-year students.
- A more simple – but also formal – test for normality can be conducted by calculating the coefficient of correlation between the residuals and their expected values under normality. **High** value of the coefficient of correlation is indicative of normality. This is a supplement to Q-Q plot.
- “Critical value” for various sample sizes are in Appendix Table B6.

When the distribution (of the response) is only near normal, **most of the dots (on the Q-Q plot" are already very close to a straight line; the "cut-point" for rejection is quite high**. Again, as mentioned, a formal statistical test may not really be needed here; but could use to supplement the Q-Q plot – more valuable when sample size n is small.

EXAMPLE: **Toluca Company Data**

(Description on page 19 of Text)

LotSize	WorkHours
80	399
30	121
50	221
90	376
70	361
60	224
120	546
80	352
100	353
50	157
40	160
70	252
90	389
20	113
110	435
100	420
30	212
50	268
90	377
110	421
30	273
90	468
40	244
80	342
70	323

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	62.3658586	26.17743389	2.382428	0.025851	8.21371106	116.518006
X Variable 1	3.57020202	0.346972157	10.28959	4.45E-10	2.85243543	4.28796861

Results:

Correlation r = .991, n = 25

Critical value = .959

(from Table B6, p.673);

Rejection when r is small!

No departure from normality

If the probability distributions of Y are not exactly normal but do not depart seriously, the sampling distributions of b_0 and b_1 would still be approximately normal with very **little effects on the level of significance of the t-test** for independence and the coverage of the confidence intervals. Even if the probability distributions of Y are far from normal, the **effects are still minimal provided that the samples sizes are sufficiently large**; i.e. the sampling distributions of b_0 and b_1 are asymptotically normal.

OMISSION OF OTHER PREDICTORS

Residuals should also be plotted against other potential independent variables – one at a time. “Time” was an earlier example in a sequential plot. If the factor under investigation is not related to the dependent and the independent variable, one would have a horizontal band of dots centered around zero which has special clustering pattern. If it is related to either the dependent or the independent variable then we would have a graph showing the residuals departing from zeros in a systematic fashion.

This is starting step in forming multiple regression models.

PROTOTYPE EXAMPLE

Age (x)	SBP (y)
42	130
46	115
42	148
71	100
80	156
74	162
70	151
80	156
85	162
72	158
64	155
81	160
41	125
61	150
75	165

Will use for Illustration

BASIC DATA DESCRIPTION

```
options ls=79;
title "SBP versus Age";
data SBP;
input age pressure;
cards;
42 130
46 115
42 148
71 100
80 156
74 162
70 151
80 156
85 162
72 158
64 155
81 160
41 125
61 150
75 165
;
```

Notes:

(1) Can use "**data lines**" instead of "cards"

(2) Good enough for smaller data sets

(3) For a larger data set, save it as "abc.dat" or "**abc.xls**" and refer to it or import it; use **PROC IMPORT** (a bit later).

Same order as in the data

DESCRIPTIVE STATISTICS

```
options ls=79;
title "Descriptive Statistics for SBP
versus Age";
data SBP;
input X Y;
 label X = 'Age'
         Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc PRINT data=SBP;
Var X Y;
run;
proc UNIVARIATE data=SBP;
run;
```

PRINT helps to check for typos

UNIVARIATE provides typical data **summaries** such as mean, range, standard deviation, etc…

More DESCRIPTIVE STATISTICS

File name

```
Proc IMPORT out=work.hw1
datafile="C:\Documents and Settings\ADCS-C381Mayo-User\Desktop\CH01PR19.xls"
DBMS=EXCEL2000 REPLACE;
GETNAMES=YES;
run;
data hw1;
set work.hw1;
run;
```

<u>Important Part:</u>

Showing HOW to read in data file (its name & location)

```
Proc MEANS data=hw1 STDERR maxdec=1;
Var x;
run;
```

Specify max # of decimal places

Request Standard Error of the Mean

```
Proc print data=hw1(obs=20) noobs;
run;
```

Suppress the observation number

MORE OPTIONS for Proc Univariate

```
options ls=79;
title "Descriptive Statistics for SBP versus Age";
data SBP;
input X Y;
  label X = 'Age'
         Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc UNIVARIATE data=SBP;
Normal;
Plots/
Plotsize = 26;
Var Y;
run;
```

NORMAL helps to test if Blood Pressure (Y) is normally distributed

PLOTS provides three useful graphs: **Stem and Leaf, Box Plot, and Q-Q Plot**.

Option **HISTOGRAM** can be added to obtain the fourth graph.

Plotsize can be changed

Similar to that used with Q-Q Plot in Regression

RESULTING GRAPHS

```
Variable=Y             Blood Pressure

               Stem Leaf                     #             Boxplot
                 16 5                        1                |
                 16 022                      3             +------+
                 15 5668                     4             *------*
                 15 01                       2             |      |
                 14 8                        1             |  +   |
                 14                                        |      |
                 13                                        |      |
                 13 0                        1             +------+
                 12 5                        1                |
                 12                                           |
                 11 5                        1                |
                 11                                           |
                 10                                           |
                 10 0                        1                |
                    ----+----+----+----+

                Multiply Stem.Leaf by 10**+1
```

There is a separate **PROC BOXPLOT** too!

```
                        Normal Probability Plot
            167.5+                                   +++    *
                 |                                *+*
                 |                         * *  *++
                 |                      **   +++
                 |                    *    ++
                 |                      +++
                 |                    ++
                 |                 +*+
                 |               +*
                 |            +++
                 |         +++ *
                 |       ++
                 |    +++
            102.5+  ++    *
                  +----+----+----+----+----+----+----+----+----+----+
                      -2        -1         0        +1        +2
```

Similar to Q-Q plot but plotting Y, not residual on vertical axis

CORRELATION
(& Scatter Diagram)

```
options ls=79;
title "Descriptive Statistics for SBP versus Age";
data SBP;
input X Y;
  label X = 'Age'
          Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc CORR data=SBP;
run;
proc plot data=SBP;
    plot y*x='*';
run;
```

Specify Notation for the graph

Proc CORR gives the coefficient of correlation r (& the p-value)

Proc **PLOT** provides the Scatter Diagram; could choose symbol to plot.

```
Simple Statistics
Variable          N         Mean        Std Dev          Sum

X                15     65.600000     15.592123     984.000000
Y                15    146.200000     19.479660    2193.000000


Simple Statistics
Variable        Minimum        Maximum     Label

X              41.000000      85.000000    Age
Y             100.000000     165.000000    Blood Pressure

 Pearson Correlation Coefficients / Prob > |R| under Ho: Rho=0 / N = 15

                          X               Y

        X              1.00000         0.56422
        Age            0.0             0.0285

        Y              0.56422         1.00000
        Blood Pressure  0.0285         0.0
```

OUTPUT

coefficient

p-value

Note: results are symmetric

SCATTER DIAGRAM

```
 Plot of Y*X.  Symbol used is '*'.
      |
  165 +                                                 *
      |
      |                                                *              *
  160 +                                                         *
      |                                             *
      |                                                        *
  155 +                                  *
      |
      |                                          *
  150 +                             *
      |   *
      |
B 145 +
l     |
o     |
o 140 +
d     |
P 135 +
r     |
e     |
s 130 +   *
s     |
u     |
r 125 + *
e     |
      |
  120 +
      |
      |
  115 +        *
      |
      |
  110 +
      |
  105 +
      |
  100 +                                           *
      |      -+-------------+-------------+-------------+-------------+-------------+-
      40            50            60            70            80            90
```

SIMPLE LINEAR REGRESSION (& Scatter Diagram)

```
options ls=79;
title "Descriptive Statistics for SBP versus Age";
data SBP;
input X Y;
 label X = 'Age'
         Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc REG data = SBP;
model y = x;
plot y*x='+';

run;
```

Proc REG is the most basic one; will add in more options

PLOT provides the Scatter Diagram; could choose symbol to plot.

Key: Model Statement

CORR and REG provide the same Scatter Diagram ("plot" option)

PARAMETER ESTIMATES

Testing for Zero Intercept
(usually not needed)

```
                      Parameter      Standard      T for H0:
Variable         DF   Estimate       Error         Parameter=0   Prob > |T|

INTERCEP          1   99.958515   19.25516927          5.191       0.0002
X (Age)           1    0.704901    0.28607866          2.464       0.0285

                      Variable
Variable         DF   Label

INTERCEP          1    Intercept
X                 1    Age
```

Slope

Testing for Zero Slope
(i.e. Independence)

ANALYSIS OF VARIANCE

Testing for Independence

```
                         Sum of         Mean
Source          DF      Squares        Square        F Value     Prob>F

Model            1   1691.19774    1691.19774          6.071     0.0285
Error           13   3621.20226     278.55402
Total           14   5312.40000

      Root MSE        16.68994    R-square      0.3183
      Dep Mean       146.20000
      C.V.            11.41583
```

From R^2 & slope, obtain “r”

MSE & its square root

USEFUL OPTIONS FROM PROC REG

- R: Analysis of residuals
- P: computing predicted values (i.e. fitted)
- COVB: Var-Cov matrix of regression coefficients
- CLM: Confidence Intervals of mean responses
- CLI: Conf Intervals of new individual responses

$$\begin{bmatrix} s^2(b_0) & s(b_0, b_1) \\ s(b_0, b_1) & s^2(b_1) \end{bmatrix}$$

ANALYSIS OF RESIDUALS

```
options ls=79;
title "Descriptive Statistics for SBP versus Age";
data SBP;
input X Y;
  label X = 'Age'
          Y = 'Blood Pressure';
cards;
42 130
…
75 165
;
proc reg data = SBP noprint;
model y = x/R;
run;
```

This short program achieves the same thing and more; it helps to set up a Table just like **TABLE 1.2 on page 22** of the text book and student residuals – plus all regression analysis results.

Standard Results come first

Analysis of Variance

```
                       Sum of         Mean
Source        DF      Squares        Square     F Value    Prob>F

Model          1   1691.19774   1691.19774       6.071     0.0285
Error         13   3621.20226    278.55402
C Total       14   5312.40000

    Root MSE       16.68994    R-square       0.3183
    Dep Mean      146.20000    Adj R-sq       0.2659
    C.V.           11.41583
```

Parameter Estimates

```
                        Parameter     Standard       T for H0:
Variable        DF      Estimate      Error          Parameter=0   Prob > |T|

INTERCEP         1     99.958515   19.25516927          5.191         0.0002
X                1      0.704901    0.28607866          2.464         0.0285
```

… then Results for Residuals

	Dep Var	Predict	Std Err		Std Err	Student
Obs	Y	Value	Predict	Residual	Residual	Residual
1	130.0	129.6	8.010	0.4357	14.642	0.030
2	115.0	132.4	7.072	-17.3839	15.118	-1.150
3	148.0	129.6	8.010	18.4357	14.642	1.259
4	100.0	150.0	4.578	-50.0065	16.050	-3.116
5	156.0	156.4	5.962	-0.3506	15.589	-0.022
6	162.0	152.1	4.934	9.8788	15.944	0.620
7	151.0	149.3	4.489	1.6984	16.075	0.106
8	156.0	156.4	5.962	-0.3506	15.589	-0.022
9	162.0	159.9	7.027	2.1249	15.139	0.140
10	158.0	150.7	4.682	7.2886	16.020	0.455
11	155.0	145.1	4.334	9.9278	16.118	0.616
12	160.0	157.1	6.163	2.9445	15.510	0.190
13	125.0	128.9	8.252	-3.8594	14.507	-0.266
14	150.0	143.0	4.506	7.0425	16.070	0.438
15	165.0	152.8	5.080	12.1739	15.898	0.766

These are Studentized Residuals

EXAMPLE: Option COVB

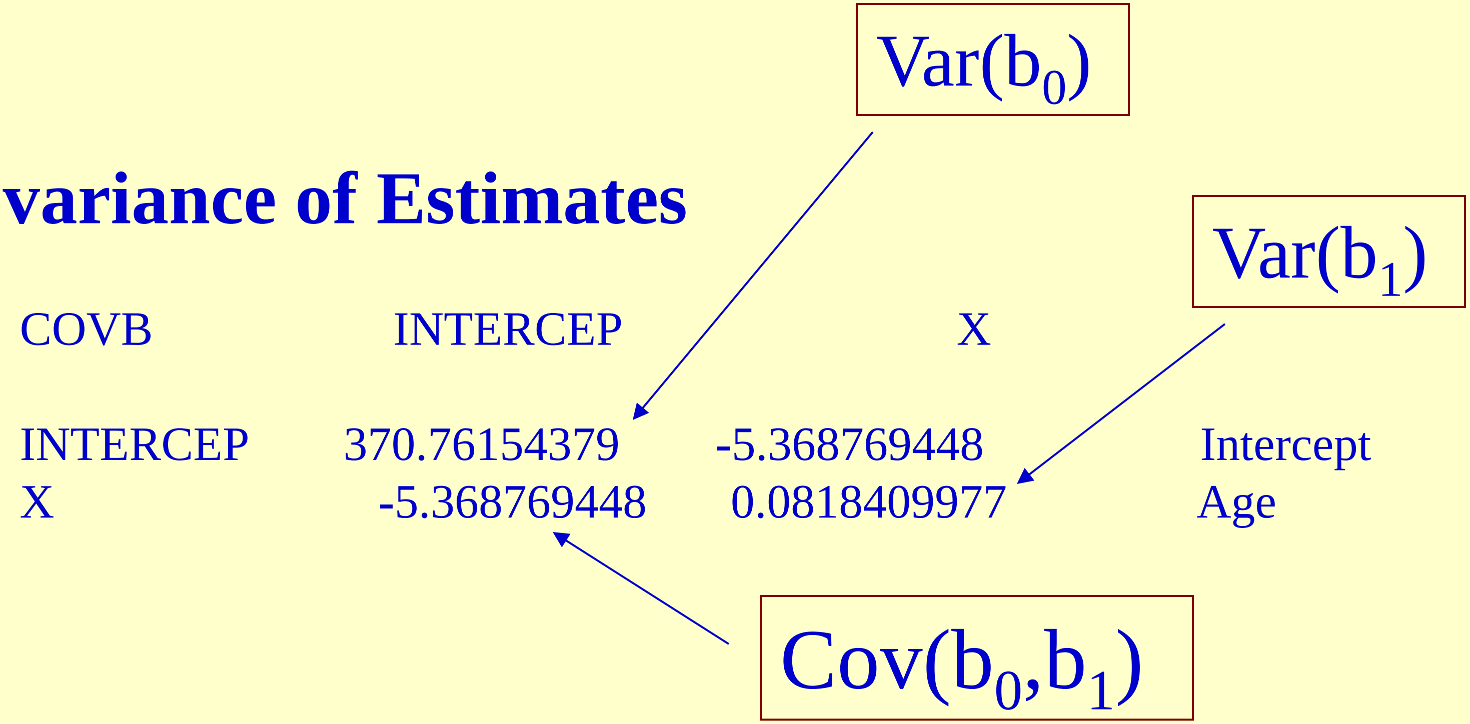
```
options ls=79;
title "Descriptive Statistics for SBP
versus Age";
data SBP;
input X Y;
  label X = 'Age'
          Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc REG data = SBP;
model y = x/COVB;
run;
```

Parameter Estimates

```
                        Parameter       Standard      T for H0:
Variable        DF       Estimate          Error   Parameter=0    Prob > |T|

INTERCEP         1      99.958515   19.25516927          5.191        0.0002
X                1       0.704901    0.28607866          2.464        0.0285
```

Covariance of Estimates



```
COVB              INTERCEP                 X

INTERCEP       370.76154379     -5.368769448            Intercept
X               -5.368769448     0.0818409977           Age
```

$\mathrm{Var}(b_0)$

$\mathrm{Var}(b_1)$

$\mathrm{Cov}(b_0,b_1)$

EXAMPLE: Option CLM

```
options ls=79;
title "Descriptive Statistics for SBP
versus Age";
data SBP;
input X Y;
  label X = 'Age'
        Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc REG data = SBP;
model y = x/CLM;
run;
```

```
          Dep Var    Predict  Std Err  Lower95%  Upper95%
  Obs           Y      Value  Predict      Mean      Mean  Residual

    1       130.0      129.6    8.010     112.3     146.9    0.4357
    2       115.0      132.4    7.072     117.1     147.7  -17.3839
    3       148.0      129.6    8.010     112.3     146.9   18.4357
    4       100.0      150.0    4.578     140.1     159.9  -50.0065
    5       156.0      156.4    5.962     143.5     169.2   -0.3506
    6       162.0      152.1    4.934     141.5     162.8    9.8788
    7       151.0      149.3    4.489     139.6     159.0    1.6984
    8       156.0      156.4    5.962     143.5     169.2   -0.3506
    9       162.0      159.9    7.027     144.7     175.1    2.1249
   10       158.0      150.7    4.682     140.6     160.8    7.2886
   11       155.0      145.1    4.334     135.7     154.4    9.9278
   12       160.0      157.1    6.163     143.7     170.4    2.9445
   13       125.0      128.9    8.252     111.0     146.7   -3.8594
   14       150.0      143.0    4.506     133.2     152.7    7.0425
   15       165.0      152.8    5.080     141.9     163.8   12.1739

Sum of Residuals                 0
Sum of Squared Residuals     3621.2023
```

EXAMPLE: Option CLI

```
options ls=79;
title "Descriptive Statistics for SBP
versus Age";
data SBP;
input X Y;
  label X = 'Age'
           Y = 'Blood Pressure';
cards;
42 130
46 115
…
75 165
;
proc REG data = SBP;
model y = x/CLI;
run;
```

Output Statistics (from CLI)

Versus: [112.3,146.9]

Under CLM

```
        Dep Var  Predicted    Std Error
Obs           Y      Value  Mean Predict      95% CL Predict        Residual
  1    130.0000   129.5643       8.0095      89.5709   169.5578      0.4357
  2    115.0000   132.3839       7.0718      93.2244   171.5435    -17.3839
  3    148.0000   129.5643       8.0095      89.5709   169.5578     18.4357
  4    100.0000   150.0065       4.5779     112.6183   187.3946    -50.0065
  5    156.0000   156.3506       5.9616     118.0630   194.6382     -0.3506
  6    162.0000   152.1212       4.9341     114.5221   189.7202      9.8788
  7    151.0000   149.3016       4.4894     111.9635   186.6396      1.6984
  8    156.0000   156.3506       5.9616     118.0630   194.6382     -0.3506
  9    162.0000   159.8751       7.0265     120.7535   198.9966      2.1249
 10    158.0000   150.7114       4.6821     113.2630   188.1598      7.2886
 11    155.0000   145.0722       4.3336     107.8201   182.3242      9.9278
 12    160.0000   157.0555       6.1628     118.6195   195.4914      2.9445
 13    125.0000   128.8594       8.2521      88.6365   169.0824     -3.8594
 14    150.0000   142.9575       4.5058     105.6102   180.3047      7.0425
 15    165.0000   152.8261       5.0795     115.1367   190.5154     12.1739
        Sum of Residuals                    0
        Sum of Squared Residuals   3621.20226
```

Note: wider Intervals

```
proc reg data=example;
model y=x/alpha=0.01 cli clm;
run;
```

New Important Part:

Set 99% CI instead of 95%

Readings & Exercises

- Readings: A thorough reading of the text’s sections 3.1-3.3 (pp. 100-114) and 3.5-3.7 (pp. 115-127) is highly recommended.
- Exercises: The following exercises are good for practice, all from chapter 3 of text: 3.3, 3.7, 3.8, 3.9, 3.10, 3.11, and 3.18.

Due As Homework

#10.1 Refer to dataset “Cigarettes”, let X=CPD and Y= log(NNAL):

a) Prepare a Box plot for log(NNAL), and from the plot: (i) Are there any points in each plot that can be considered as extreme?, and (ii) Does this plot look symmetric? (the result may explain why we use NNAL on log scale)

b) Plot the residuals against predictor’s values; What departures from the Normal Regression Model can be studied from this plot? What are your findings?

c) Conduct the Brown-Forsythe test to determine whether or not the error variance varies with the level of X

#10.2 Answer the 3 questions of Exercise 10.1 using dataset “Infants” with X = Gestational Weeks and Y = Birth Weight.