PubH 7401: Elements of Biostatistical Inference I – 2008
Homework 5, due Tuhrsday, October 9

1. At a particular restaurant, the price of an entree is $E \sim N(10, 4)$ in U.S. dollars. That is, E is normally distributed with $\mu = 10$ and $\sigma^2 = 4$. Dessert is $D \sim N(4, 1)$ and a glass of wine is $W \sim N(6, 2)$.

   (a) What is the distribution of the total cost of dinner T for one at this restaurant, assuming independence among E, D, and W? Also assume that only one entree, one dessert, and one glass of wine will be consumed.

   (b) If I only have \$10, what is the probability that I can afford dinner? That is, find $P(T \leq 10)$.

2. Let (X, Y) be jointly distributed with pdf

$$f(x,y) = \begin{cases} 4xy & \text{for } \begin{matrix} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{matrix} \\ 0 & \text{otherwise} \end{cases}$$

   (a) Does the joint pdf factor into $f(x, y) = g(x)h(y)$? If so, what are these functions?

   (b) Are X and Y independent? Why or why not?

   (c) Find the conditional pdf's $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$. HINT: If X and Y are independent, this is easy...

3. Let (X, Y) be jointly distributed with pdf

$$f(x,y) = \begin{cases} x+y & \text{for } \begin{matrix} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{matrix} \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the marginal pdf's $f_X(x)$ and $f_Y(y)$.

   (b) Find the conditional pdf's $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.

   (c) Are X and Y independent? Why or why not?

   (d) Find $P(X > Y)$.

4. Let X_1, X_2, X_3, X_4 be independent random variables, each having distribution $f(x) = \frac{8}{3}x^3 \exp(-2x)$ for $x \geq 0$.

   (a) What kind of density is this? e.g. $X_i \sim$?

   (b) Find the joint pdf of (X_1, X_2, X_3, X_4), namely $f(x_1, x_2, x_3, x_4)$.

5. Let (X, Y) be jointly distributed with the pmf

x	y	$p(x, y)$
2	1	0.1
2	2	0.2
4	2	0.1
6	3	0.4
8	3	0.2

   (a) Find $p_Y(y) = P(Y = y)$ and $p_X(x) = P(X = x)$.

   (b) What is $p_{X|Y}(x|2) = P(X = x|Y = 2)$ for $x \in \{2, 4, 6, 8\}$?

   (c) What is $P(XY \leq 8)$?

   (d) Are X and Y independent? Why or why not?

6. Use the pmf $p(x, y)$ for discrete (X, Y) in Chapter 3, Problem 1 (p. 107) to answer the following questions.

   (a) What is $P(X = Y^2)$?

   (b) What is $P(X = 1)$?

   (c) What is $P(X = 1|Y = 4)$?

   (d) Can X and Y be independent? Why or why not?

   (e) Let $Z = X + Y$. What is the range of Z? What is $P(Z = 5)$?

7. Use the joint density $f(x, y)$ for continuous (X, Y) in Chapter 3, Example A (p. 75, bottom) to answer the following questions.

   (a) Showing all work, verify that $f_Y(y) = \frac{12}{7}\left(\frac{1}{3} + \frac{y}{2}\right)$.

   (b) Find $f_{X|Y}(x|y)$.

   (c) From Example B (p. 76), $f_X(x) = \frac{12}{7}\left(x^2 + \frac{x}{2}\right)$. Does $f_X(x) = f_{X|Y}(x|y)$? That is, does X care about Y?

   (d) Can X and Y be independent? Why or why not?