Homework 1 Solution

2. a). Sample Space

$\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}.$

b). $A=\{(1,4)\ (1,5)\ (1,6)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}.$

$B=\{(2,1)\ (3,1)\ (4,1)\ (5,1)\ (6,1)$
$(3,2)\ (4,2)\ (5,2)\ (6,2)$
$(4,3)\ (5,3)\ (6,3)$
$(5,4)\ (6,4)\ (6,5)\}.$

$C=\{(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)\}.$

c) $A\cap C = C$

$B\cap C=\{(2,1)\ (3,1)\ (3,2)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\}$

$A\cap(B\cup C)=\{(3,2)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)\ (5,1)\ (5,2)$
$(5,3)\ (5,4)\ (6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\}.$

14. 52 cards randomly be divided into 4 groups each has 13 cards. So there are

$$\frac{52!}{13!\,13!\,13!\,13!} = \binom{52}{13}\binom{39}{13}\binom{26}{13} \text{ possibilities}$$

16. The problem could be interpreted as 13 positions divided into 7 groups, each has 1, 1, 2, 2, 2, 2, 3 positions.

$$\frac{13!}{2!2!2!2!\,3!}$$ possibilities.

20. There are 52! ways to arrange the cards.
If we bind the 4 Aces together and view them as 1 card, there are 49! ways to arrange the cards, and within the 4 Aces, there are 4! ways to arrange the 4 Aces.
So the probability of 4 Aces together is:

$$\frac{4!\,49!}{52!}$$

30. A. To devide all the students into 2 classes, there are

$$\frac{60!}{30!\,30!}$$ ways.

If the five are in the same class, suppose they are all in class A, then we only have to divide the rest 55 students into 2 classes, with 25 students in class A and 30 in class B. We have $\frac{55!}{25!\,30!}$ ways to do that.

And the five students can also be all in class B. So the probability of the five all in one class is:

$$2 \times \frac{55!}{25!\,30!} \Big/ \frac{60!}{30!\,30!}$$

B. First we have to pick 4 students out of the 5 students and put them in class A and the other one in class B. Then we divide the rest 55 students into 2 classes with 26 in class A and 29 in class B. Also, the 4 students could be in class B and the other one in class A. So the probability of 4 of them in one class and the other one in another class is $2 \times \frac{55!\binom{5}{4}}{26!\,29!} \Big/ \frac{60!}{30!\,30!}$

c. This time we don't have to pick the 4 students from the 5 students. So the probability is. $2 \times \frac{55!}{26!29!} \Big/ \frac{60!}{30!30!}$

38. ① This problem is interpreted as 6 positions divided into 2 groups each has 3 positions. So there are $6!/3!3!$ ways.
② Similarly, we have $\frac{9!}{3!3!3!}$ ways.

42. 11 boys are divided into 4 groups, each has 4, 3, 3, 1 players.
So the possibilities are $\frac{11!}{4!3!3!}$

46. ① If you toss a "H", then you'll have ~~[illegible]~~ $\frac{3}{5}$ chance to get a red ball.
If you toss a "T", then you'll have $\frac{2}{7}$ chance to get a red ball. So the probability to get a red ball is.
$P(\text{red ball}) = \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{7} = \frac{3}{10} + \frac{1}{7} = \frac{31}{70} = 0.44$
② Use Bayes Rule.

$$P(H \mid \text{red}) = \frac{P(\text{red} \mid H)P(H)}{P(\text{red} \mid H)P(H) + P(\text{red} \mid T)P(T)} = \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times (\frac{3}{6} + \frac{2}{7})}$$

$$= \frac{21}{155} = 0.135$$

50 $P(\text{at least one is } 3 \mid \text{sum} = 6)$.

$$= \frac{P(\text{at least one is } 3, \text{sum} = 6)}{P(\text{sum} = 6)}.$$

$\{\text{at least one is } 3, \text{Sum} = 6\} = \{(3,3)\}$.

$\{\text{sum} = 6\} = \{(1,5)\ (2,4)\ (3,3).\ (4,2)\ (5,1)\}$.

$P(\text{at least one is } 3) = 1/5$.

56 ① P(both of the children are girls)

$= P($ the oldest one is a girl and the youngest is also girl$)$

$= P($the oldest is girl$)P($the youngest is girl$)$

$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P(oldest one is girl) $= \frac{1}{2}$.

~~So P(old is girl)~~

So P(both are girls | oldest is girl) $= \dfrac{P(\text{both are girls})}{P(\text{oldest is girl})}$

$= \frac{1}{4} / \frac{1}{2} = \frac{1}{2}$

② P (one of the children is girl)

$= 1 - P(\text{both are boys}) = 1 - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$

So, P (both are girls | at least one is girl)

$= \dfrac{P(\text{both are girls})}{P(\text{at least one girl})} = \frac{1}{4} / \frac{3}{4} = \frac{1}{3}$

60 ① the percentage of defectivity is

P(an item is defective)

$= P(\text{defective} \mid \text{made by } 1)\, P(\text{made by } 1)$

$+ P(\text{defective} \mid \text{made by } 2)\, P(\text{made by } 2)$

$+ P(\text{defective} \mid \text{made by } 3)\, P(\text{made by } 3)$

$= \frac{1}{3}(1\% + 2\% + 5\%) = \frac{8}{300} = 0.0267$

② P(made by 3| defective)

$= \dfrac{P(\text{defective} \mid \text{made by } 3)P(\text{made by } 3)}{P(\text{defective} \mid \text{made by } 3)P(\text{made by } 3) + P(\text{defective} \mid \text{not } 3)P(\text{not } 3)}$

$= \dfrac{\frac{5}{300}}{\frac{5}{300} + \frac{1+2}{300}} = \frac{5}{8} = 0.625$

70. If B = Sample Space $\quad P(B)=1$

$P(A\cap B) = P(A) = P(A)\cdot P(B)$

If A = empty Space. $\quad P(A)=0$.

$P(A\cap B) = P(\phi) = 0 = P(A)P(B)$.

So. A and B are independent in this situation.

If A and B are not ϕ or Sample Space, then if $A\subset B$, they can't be independent.

78. a.

	A	A
A	$\frac{1}{2}\times\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{2}$
a	$\frac{1}{2}\times\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{2}$

$P(AA) = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$

$P(Aa) = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$

b. Let P_A = Probability of allele type A in the 1 generation.

P_a = Probability of allele type a in the 1 generation.

Since probability of AA, Aa, aa are p, q, r,

Therefore $P_A = \dfrac{2\times p + 1\times 2q}{2\times(p+2q+r)} = p+q$.

$P_a = \dfrac{1\times 2q + 2\times r}{2\times(p+2q+r)} = q+r$

So in the second generation,

$P_2(AA) = P_A^2 = (p+q)^2$

$P_2(Aa) = 2P_A P_a = 2(p+q)(q+r)$

$P_2(aa) = P_a^2 = (q+r)^2$

$P_{A2} = \dfrac{2\times(p+q)^2+2(p+q)(q+r)}{2} = (p+q)(1+q+r) = (p+q)$

$P_{a2} = \dfrac{2\times(q+r)^2+2(p+q)(q+r)}{2} = (q+r)(1+p+q) = (q+r)$

(Because $p+2q+r=1$)

So in the third generation

$P_3(AA) = P_{A_2}^2 = (p+q)^2 = P_2(AA)$.

$P_3(Aa) = 2P_{A_2}P_{a_2} = 2(p+q)(q+r) = P_2(Aa)$

$P_3(aa) = P_{a_2}^2 = (q+r)^2 = P_2(aa)$.

c. The probability of AA, Aa, aa, in the 1st generation after they survive to mate is

$P_1(AA) = pu$. $P_1(Aa) = 2qv$ $P_1(aa) = rw$.

and $pu + qv + rw = 1$.

So $p_A = \frac{2pu + 2qv}{2} = pu + qv$.

$p_a = \frac{2qv + 2rw}{2} = qv + rw$.

Therefore in the second generation

$P_2(AA) = (pu+qv)^2$

$P_2(Aa) = 2(pu+qv)(qv+rw)$.

$P_2(aa) = (qv+rw)^2$

in the third generation

$P_3(AA) = P_2(AA)$

$P_3(Aa) = P_2(Aa)$

$P_3(aa) = P_3(aa)$