# Solution of HW 3.

2.52. Let $X$ = individual's height in inches.

$X \sim N(70, 9)$.

a). $P(X > 72) = P\left(\frac{X-70}{3} > \frac{72-70}{3}\right) = \Phi\left(\frac{2}{3}\right) = 0.748$.

b). $Y$ = individual's height in centimeters.

$Y = 2.54X \sim N(70 \times 2.54,\ 9 \times 2.54^2) = N(177.8,\ 58.06)$.

$Z$ = individual's height in meters.

$Z = \frac{1}{100}Y \sim N\left(\frac{1}{100} \times 177.8,\ \frac{1}{100^2} \times 58.06\right) = N(1.778,\ 0.005806)$.

2.54. $P(|X-\mu| \le 0.675\sigma) = P\left(\frac{|X-\mu|}{\sigma} \le 0.675\right)$

$= P\left(-0.675 \le \frac{X-\mu}{\sigma} \le 0.675\right) = \Phi(0.675) - \Phi(-0.675) = 0.5$.

2.56 $F_Y(y) = P(Y \le y) = P(|X| \le y) = P(-y \le X \le y)$.

$= P\left(-\frac{y}{\sigma} \le \frac{X}{\sigma} \le \frac{y}{\sigma}\right) = \Phi\left(\frac{y}{\sigma}\right) - \Phi\left(-\frac{y}{\sigma}\right) \quad (y \ge 0)$

$F_Y(y) = 0 \quad (y < 0)$.

$f_Y(y) = F_Y'(y) = \frac{1}{\sigma}\Phi'\left(\frac{y}{\sigma}\right) + \frac{1}{\sigma}\Phi'\left(-\frac{y}{\sigma}\right)$

$= \frac{2}{\sqrt{2\pi}\,\sigma^2} e^{-\frac{y^2}{2\sigma^4}} \quad (y \ge 0)$

2.58 Let $Y = \sqrt{U} \Rightarrow U = Y^2$

$f_Y(y) = \left|\frac{d}{dy} y^2\right| \cdot f_X(y^2) = 2y \cdot 1 = 2y \quad (1 \ge y \ge 0)$

2.64 $V \sim N(0, \sigma^2)$

$E = \frac{1}{2} m V^2 \Rightarrow |V| = \sqrt{\frac{2E}{m}}$

$\therefore F_E(y) = P(E \le y) = P(\frac{1}{2} m V^2 \le y) = P(|V| \le \sqrt{\frac{2y}{m}})$.

$= P(-\sqrt{\frac{2y}{m\sigma^2}} \le \frac{V}{\sigma} \le \sqrt{\frac{2y}{m\sigma^2}})$

$= \Phi(\sqrt{\frac{2y}{m\sigma^2}}) - \Phi(-\sqrt{\frac{2y}{m\sigma^2}}) \quad (y \ge 0) \qquad F_E(y) = 0 \quad (y < 0)$.

$f_E(y) = F_E'(y) = 2 \cdot \sqrt{\frac{2}{m\sigma^2}} \cdot \frac{1}{2} y^{-\frac{1}{2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{2y/m\sigma^2}{2\sigma^2}}$

$= \frac{1}{\sqrt{m\pi\sigma^2}} y^{-\frac{1}{2}} e^{-\frac{y}{m\sigma^4}} \qquad (y \ge 0)$

2.68. $S = \pi R^2 \Rightarrow R = \sqrt{\frac{S}{\pi}}$

$F_S(y) = P(S \le y) = P(\pi R^2 \le y) = P(R \le \sqrt{\frac{y}{\pi}})$

$= 1 - e^{-\lambda\sqrt{\frac{y}{\pi}}} \qquad (y \ge 0) \qquad F_S(y) = 0 \quad (y < 0)$.

$f_S(y) = F_S'(y) = \sqrt{\frac{1}{\pi}} \cdot \frac{1}{2} y^{-\frac{1}{2}} \lambda e^{-\lambda\sqrt{\frac{y}{\pi}}}$

$= \frac{\lambda}{2\sqrt{\pi}} y^{-\frac{1}{2}} e^{-\lambda\sqrt{\frac{y}{\pi}}} \qquad (y \ge 0)$