PubH 8492, Spring 2018 Homework #1 (23 Jan 2018)

1 (a) For the Bayesian analysis of the mixed linear model, derive the marginal posterior of the unknowns in the covariance matrices R and G, i.e., ϕ_R and ϕ_G, for a general prior on them, $\pi(\phi_R, \phi_G)$. Assume the prior on β is $\pi(\beta) \propto 1$, i.e., flat.

Hint: If it seems hard, you're taking the wrong approach.

(b) Use the foregoing to derive the marginal posterior (also the restricted likelihood) for ϕ_R, ϕ_G for the balanced, one-way random-effect model (BOWREM):

$$y_{ij} = \beta_0 + u_i + \varepsilon_{ij} \quad \text{for } i=1,\dots,N,\ j=1,\dots,m,\quad u_i \sim N(0,\sigma_s^2),\quad \varepsilon_{ij} \sim N(0,\sigma_e^2)$$

(The restricted likelihood is equation 15.11 on p. 324 of Hodges 2014.)

2 For the general model in item 1 (a) above, assume $R = \sigma_e^2 I_n$ and assume σ_e^2 has an inverse-gamma prior distribution. Re-parameterize from (σ_e^2, G) to (σ_e^2, G^*) for $G^* = \frac{1}{\sigma_e^2} G$. Assume σ_e^2 and G^* (!) are independent a priori, so that $\pi(\sigma_e^2, G^*) = \pi(\sigma_e^2)\,\pi(G^*)$. Starting from the marginal posterior for (ϕ_R, ϕ_G) in 1 (a), derive the marginal posterior distribution of G^*.

Hint: (i) Same as above.

(ii) This is not quite a trick question.

Due Tue 30 Jan 2018