

**PubH 8492-001**

**Theories of Hierarchical and Other Richly Parametrized Linear Models  
Spring 2020**

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<b>Credits:</b>	<b>3</b>
<b>Meeting Days:</b>	<b>T/Th</b>
<b>Meeting Time:</b>	<b>1:00 PM to 2:15 PM</b>
<b>Meeting Place:</b>	<b>Mayo Building Room A434</b>
<b>Instructor:</b>	<b>James S. Hodges, Professor, Division of Biostatistics</b>
<b>Office Address:</b>	<b>University Office Plaza, 2221 University Avenue SE, Suite 200, Room 224</b>
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<b>Office Hours:</b>	<b>By appointment; e-mail is the best way to contact me.</b>

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**I. Course Description**

Linear richly-parameterized models include hierarchical models, dynamic linear models, linear mixed models, random regressions, smoothers (spatial and otherwise), longitudinal models, time series models, and many others. Existing theories are mainly schemes for specifying and fitting many such models. This course's first part describes the mixed-linear-model scheme (or syntax), reviews standard theory and computing for that scheme, and briefly reviews two other schemes. Apart from this, richly-parameterized models have nothing like the deep, powerful theory of ordinary linear models. This course's second part begins with the little theory that exists, then explores odd, surprising, or undesirable results that arise routinely in applying these models to real datasets. The purpose is to illustrate the problems a theory must address, emphasizing open questions and possible approaches to solving them. Class sessions are lectures by the instructor except that each student will present one lecture that they choose. The grade depends mostly on a final class project, which can be on any topic the instructor and student agree on, preferably related to their dissertation or research-assistant work. Students will present their projects in class and hand in a written version.

**II. Course Prerequisites**

Stat 8101/8102 (or its equivalent), and Stat 8311 or PubH 8400 (Linear Models), and Biostatistics or Statistics PhD student, or permission of instructor.

**III. Course Goals and Objectives**

Present a theoretical approach to the large class of richly-parameterized linear models. Survey the beginnings of a theory of these models using datasets mostly from the instructor's collaborative projects to describe unsolved and partly-solved problems and approaches to working those problems.

#### IV. Methods of Instruction and Work Expectations

Class sessions consist of lectures by the instructor and one lecture by each student, and presentations of students' class projects. Students' grades depend on homework exercises but mostly on the final class project presented orally (in class) and in writing. Each student selects the project's topic, subject to the instructor's approval and preferably related to their dissertation or research-assistant work. Students are expected to read extensively from the course texts and from assigned journal articles. All materials for this course are on the instructor's web page (<http://www.biostat.umn.edu/~hodges/>, third item under "Links").

#### V. Course Text and Readings

Required text: Ruppert D, Wand MP, Carroll RJ (2003). *Semiparametric Regression*. Cambridge U Press

Supplementary texts: Hodges JS (2014). *Richly Parameterized Linear Models: Additive, Time Series, and Spatial Models Using Random Effects*, Chapman and Hall; Rue H, Held L (2005). *Gaussian Markov Random Fields: Theory and Applications*, Chapman and Hall; Lee Y, Nelder JA, Pawitan Y (2006). *Generalized Linear Models with Random Effects: Unified Analysis via H-likelihood*, Chapman and Hall.

Other readings: These include published or submitted journal articles, with availability as described below.

Available on the course web page:

Cui Y, Hodges JS, Kong X, Carlin BP (2010). Partitioning degrees of freedom in hierarchical and other richly-parameterized models. *Technometrics*, 52:124-136.

Hodges JS (1998). Some algebra and geometry for hierarchical models, applied to diagnostics (with discussion). *Journal of the Royal Statistical Society, Series B*, 60:497-536.

Hodges JS, Clayton MK (2011). Random effects old and new. Unpublished manuscript.

Hodges JS, Cui Y, Sargent DJ, Carlin BP (2007). Smoothing balanced, single-error-term analysis of variance. *Technometrics*, 49:12-25.

Hodges JS, Reich BJ (2010). Adding spatially-correlated errors can mess up the fixed effect you love. *The American Statistician*, 64:325-334.

Martinez-Beneito MA, Hodges JS, Marí-Dell-Olmo M (pre-publication version, 2014). Smoothed ANOVA Modeling. *CRC Handbook of Spatial Epidemiology*, eds. Andrew B Lawson, Sudipto Banerjee, Robert Haining, Lola Ugarte, to appear.

Reich BJ, Hodges JS (2008). Identification of the variance components in the general two-variance model. *Journal of Statistical Planning and Inference*, 138:1592-1604.

Reich BJ, Hodges JS, Carlin BP (2007). Spatial analyses of periodontal data using conditionally autoregressive priors having two classes of neighbor relations. *Journal of the American Statistical Association*, 102:44-55.

Reich BJ, Hodges JS, Zadnik V (2006). Effects of residual smoothing on the posterior of the fixed effects in disease-mapping models. *Biometrics*, 62:1197-1206.

Available from the instructor in hard-copy only.

Lavine M, Hochman D, Haglund D (2009). Optical images of the brain during surgery: Detection of hemodynamics in response to electrical stimulus. Unpublished manuscript.

#### VI. Course Outline/Weekly Schedule

Below, H2014 refers to Hodges (2014), "RWC" refers to Ruppert, Wand, and Carroll (2003), LNP refers to Lee, Nelder, and Pawitan (2006), and RH refers to Rue and Held (2005), each cited in Section V above as a required or supplementary text. Journal articles are cited in standard format, e.g., "Reich et al (2006)", referring to the list of readings given in Section V above.

Weeks 1 through 7 are Part I of the course, covering syntaxes for richly parameterized models, focusing on the mixed linear model syntax and associated inferential tools. Weeks 8 through 14 are Part II of the course, showing the beginnings of a full theory of richly-parameterized models mostly by showing odd, mysterious, or incorrect results produced by standard analyses and developing theory to explain them. Homework assignments are concentrated in Part I of the course; during Part II, it is expected that students will work on

their final class projects. The instructor will meet at least twice with each student regarding her/his final class project, or three times if enrollment is low enough to permit it.

Week 1/ January 21, 23. One or two introductory examples intended to scare you, about how much anybody really understands even relatively simple models. The mixed linear model in the standard form, and the conventional (non-Bayesian) analysis of it.

Readings: H2014 Chapter 1 (sections 1.1, 1.2), RWC chapters 1, 2 (through 2.6), 4 (except 4.9).

Week 2/ January 28, 30. Bayesian analysis of the mixed linear model; pros and cons of non-Bayesian and Bayesian analyses, and problems with each approach.

Readings: H2014 Sections 1.3-1.5, RWC chapter 16.

1<sup>st</sup> homework assignment handed out on Tuesday 28 January, due Tuesday 4 February.

Week 3/ February 4, 6. An alternative formulation of the mixed linear model (the constraint-case formulation). Measures of complexity in a mixed linear model fit (degrees of freedom).

Readings: Required: H2014 Chapter 2; if you're keen on degrees of freedom, Cui et al (2010).

2<sup>nd</sup> homework assignment handed out on Thu 6 February, due Thu 13 February.

Week 4/ February 11, 13. Smoothing using penalized splines: bases, knots, penalized splines, rank of smoothers; penalized splines as mixed linear models: fitted values, inference, complexity of the fit (degrees of freedom).

Reading: H2014 Chapter 3; RWC Chapter 3, Chapter 4 section 9, Chapter 6 sections 1-4. Optional: RWC Chapter 6 sections 5-9.

Week 5/ February 18, 20. Additive models and models with interactions, represented as mixed models; using priors on degrees of freedom to control the extent of smoothing.

Reading: H2014 Chapter 4; RWC Chapters 7, 8, 9, and 12 (sections 1-3). If you're keen on smoothed ANOVA, Hodges et al (2007); if you're keen to see spatial smoothing in the context of smoothed ANOVA, Martinez-Beneito et al (2014).

3<sup>rd</sup> homework assignment handed out on Tue 18 Feb, due Tue 25 Feb.

Week 6/ February 25, 27. Spatial smoothing using mixed linear models: smoothing on a lattice using improper conditional autoregressive (CAR) models; two-dimensional splines, mostly with radial basis functions.

Reading: H2014 Chapter 5; RWC Chapter 13, sections 1-4.

Week 7/ March 3, 5. Time series using dynamic linear models (Kalman filters). Quick looks at the alternative syntaxes in RH and LNP. Summary of Part I of the course.

Reading: H2014 Chapters 6, 7. Optional: Browse RH or LNP.

4th homework assignment handed out on Tue 25 February, due Thu 5 March. This assignment is a proposal for the student's final class project; each student will meet individually with the instructor during week 8 to discuss the proposal.

Spring Break/ 9-13 March, no classes

Week 8/ March 17, 19. Introduction to Part II: what do we want in a theory of richly parameterized models? From linear models to richly-parameterized models: Mean structure. Generalizations of diagnostics from single-variance linear models, using the constraint-case formulation: Some diagnostics generalize, some don't, and we begin to see what's different about richly-parameterized models.

Reading: H2014 Chapter 8. If you'd enjoy seeing your professor embarrassed in print, read Hodges (1998) especially Section 5.2 and Wakefield's discussion.

Final class project: First meeting with the instructor about the final class project.

Week 9/ March 24, 26. Continuing the previous week's material. Beginning: Collinearity and confounding in the presence of smoothing/shrinkage. Teaser: Weird things that happened fitting richly-parameterized models in four real applications. First weird thing: Adding a spatial random effect to a model makes an

obvious association disappear (spatial confounding); mechanics of how it happens, how to interpret it, and what to do about it.

Reading: H2014 Chapter 9 (teasers), Section 10.1; if you're keen to read the original papers about spatial confounding, Reich et al (2006), Hodges & Reich (2010).

Final class project: Students are expected to be working on their final class projects.

Week 10/ March 31, April 2. Continuing the previous week's material

Reading: Same as previous week's

Final class project: 2<sup>nd</sup> meeting with the instructor about the final class project.

Week 11/ April 7, 9. Third weird thing: Adding a random effect radically changes the fit of two other random effects (i.e., collinear or competing random effects). Generating and testing hypotheses about why this happens.

Reading: H2014, Chapter 12; Lavine et al (2009).

Final class project: Students are expected to be working on their final class projects.

Week 12/ April 14, 16. Traditional random effects (as in, e.g., Scheffé's classic ANOVA text) versus new-style random effects (i.e., most random effects discussed in this course), which have the mathematical form of a traditional random effect but which do not meet the traditional definition. Teaser for the last part of the course: Weird things related to estimation of unknowns in the variance structure.

Reading: Random effects old and new: H2014 Chapter 13. Teaser: Weird things ..., H2014 Chapter 14.

Final class project: 3<sup>rd</sup> meeting with the instructor about the final class project.

Week 13/ April 21, 23. Beyond linear models: Variance structure. The general 2-variance model: Re-expressing the restricted likelihood as the likelihood from a particular generalized linear model, which enables many existing tools for examining how the data provide information about the two variances. Examples; application to some puzzles described in the previous week's Teasers.

Reading: H2014, Chapters 15, 16.

Final class project: Students are expected to be working on their final class projects.

Week 14/ April 28, 30. Extending this approach to more complex models; two expedients of possible use when it can't be extended. Applications to more puzzles from the Teasers. A bit on estimates on the boundary of the parameter space and on restricted likelihoods and posterior distributions with multiple maxima. In-class presentation of student projects.

Reading: H2014, Chapters 17 (you can skip the gruesome details), 18, 19 (the latter two are short).

Final class project: Students will do a formal (ENAR-style) in-class presentation of their final class projects, and are expected to be working on the written version that's due at the end of exam week.

Exam week: IF NECESSARY (but it hasn't been, the last few offerings), one or two sessions (depending on class size and students' schedules) of oral presentations of final class projects. Written version of the final class project is due Monday 13 May (the last day of exams).

## VII. Evaluation and Grading

Grading will be A-F. 20% of the grade will be based on homework exercises, 60% will be based on the written version of the final class project; 20% will be based on the oral presentation of the final class project.

**For additional information, please refer to** the University's Uniform Grading Policy and Grading Rubric Resource at <https://z.umn.edu/gradingpolicy>.

You are expected to do your own academic work and cite sources as necessary. Failing to do so is scholastic dishonesty. Scholastic dishonesty means plagiarizing; cheating on assignments or examinations; engaging in unauthorized collaboration on academic work; taking, acquiring, or using test materials without faculty permission; submitting false or incomplete records of academic achievement; acting alone or in cooperation with another to falsify records or to obtain dishonestly grades, honors, awards, or professional endorsement; altering, forging, or misusing a University academic record; or fabricating or falsifying data,

research procedures, or data analysis (As defined in the Student Conduct Code). For additional information, please see <https://z.umn.edu/dishonesty>.

The Office for Student Conduct and Academic Integrity has compiled a useful list of Frequently Asked Questions pertaining to scholastic dishonesty: <https://z.umn.edu/integrity>.

If you have additional questions, please clarify with your instructor. Your instructor can respond to your specific questions regarding what would constitute scholastic dishonesty in the context of a particular class- e.g., whether collaboration on assignments is permitted, requirements and methods for citing sources, if electronic aids are permitted or prohibited during an exam.

Indiana University offers a clear description of plagiarism and an online quiz to check your understanding (<http://z.umn.edu/iuplagiarism>).

### **Incomplete Contracts**

A grade of incomplete "I" shall be assigned at the discretion of the instructor when, due to extraordinary circumstances (e.g., documented illness or hospitalization, death in family, etc.), the student was prevented from completing the work of the course on time. The assignment of an "I" requires that a contract be initiated and completed by the student before the last official day of class, and signed by both the student and instructor. If an incomplete is deemed appropriate by the instructor, the student in consultation with the instructor, will specify the time and manner in which the student will complete course requirements.

Extension for completion of the work will not exceed one year (or earlier if designated by the student's college). For more information and to initiate an incomplete contract, students should go to SPHGrades at: [www.sph.umn.edu/grades](http://www.sph.umn.edu/grades).

## **VIII. SPH AND UNIVERSITY POLICIES & RESOURCES**

The School of Public Health maintains up-to-date information about resources available to students, as well as formal course policies, on our website at [www.sph.umn.edu/student-policies/](http://www.sph.umn.edu/student-policies/). Students are expected to read and understand all policy information available at this link and are encouraged to make use of the resources available.

The University of Minnesota has official policies, including but not limited to the following:

- Grade definitions
- Scholastic dishonesty
- Makeup work for legitimate absences
- Student conduct code
- Sexual harassment, sexual assault, stalking and relationship violence
- Equity, diversity, equal employment opportunity, and affirmative action
- Disability services
- Academic freedom and responsibility

Resources available for students include:

- Confidential mental health services
- Disability accommodations
- Housing and financial instability resources
- Technology help
- Academic support

## **VIII. Other Course Information and Policies**

### **Course Withdrawal**

Students should refer to the Refund and Drop/Add Deadlines for the particular term at [onestop.umn.edu](http://onestop.umn.edu) for information and deadlines for withdrawing from a course. As a courtesy, students should notify their instructor and, if applicable, advisor of their intent to withdraw.

Students wishing to withdraw from a course after the noted final deadline for a particular term must contact the School of Public Health Office of Admissions and Student Resources at [sph-ssc@umn.edu](mailto:sph-ssc@umn.edu) for further information.

### **Student Conduct Code**

The University seeks an environment that promotes academic achievement and integrity, that is protective of free inquiry, and that serves the educational mission of the University. Similarly, the University seeks a community that is free from violence, threats, and intimidation; that is respectful of the rights, opportunities, and welfare of

students, faculty, staff, and guests of the University; and that does not threaten the physical or mental health or safety of members of the University community.

As a student at the University you are expected adhere to Board of Regents Policy: *Student Conduct Code*. To review the Student Conduct Code, please see:

[http://regents.umn.edu/sites/default/files/policies/Student\\_Conduct\\_Code.pdf](http://regents.umn.edu/sites/default/files/policies/Student_Conduct_Code.pdf).

Note that the conduct code specifically addresses disruptive classroom conduct, which means "engaging in behavior that substantially or repeatedly interrupts either the instructor's ability to teach or student learning. The classroom extends to any setting where a student is engaged in work toward academic credit or satisfaction of program-based requirements or related activities."

### **Use of Personal Electronic Devices in the Classroom**

Using personal electronic devices in the classroom setting can hinder instruction and learning, not only for the student using the device but also for other students in the class. To this end, the University establishes the right of each faculty member to determine if and how personal electronic devices are allowed to be used in the classroom. For complete information, please reference:

<http://policy.umn.edu/Policies/Education/Education/STUDENTRESP.html>.

### **Makeup Work for Legitimate Absences**

Students will not be penalized for absence during the semester due to unavoidable or legitimate circumstances. Such circumstances include verified illness, participation in intercollegiate athletic events, subpoenas, jury duty, military service, bereavement, and religious observances. Such circumstances do not include voting in local, state, or national elections. For complete information, please see:

<http://policy.umn.edu/Policies/Education/Education/MAKEUPWORK.html>.

### **Appropriate Student Use of Class Notes and Course Materials**

Taking notes is a means of recording information but more importantly of personally absorbing and integrating the educational experience. However, broadly disseminating class notes beyond the classroom community or accepting compensation for taking and distributing classroom notes undermines instructor interests in their intellectual work product while not substantially furthering instructor and student interests in effective learning.

Such actions violate shared norms and standards of the academic community. For additional information, please see: <http://policy.umn.edu/Policies/Education/Education/STUDENTRESP.html>.

### **Sexual Harassment**

"Sexual harassment" means unwelcome sexual advances, requests for sexual favors, and/or other verbal or physical conduct of a sexual nature. Such conduct has the purpose or effect of unreasonably interfering with an individual's work or academic performance or creating an intimidating, hostile, or offensive working or academic environment in any University activity or program. Such behavior is not acceptable in the University setting. For additional information, please consult Board of Regents Policy:

<http://regents.umn.edu/sites/default/files/policies/SexHarassment.pdf>

### **Equity, Diversity, Equal Opportunity, and Affirmative Action**

The University will provide equal access to and opportunity in its programs and facilities, without regard to race, color, creed, religion, national origin, gender, age, marital status, disability, public assistance status, veteran status, sexual orientation, gender identity, or gender expression. For more information, please consult Board of Regents Policy: [http://regents.umn.edu/sites/default/files/policies/Equity\\_Diversity\\_EO\\_AA.pdf](http://regents.umn.edu/sites/default/files/policies/Equity_Diversity_EO_AA.pdf).

### **Disability Accommodations**

The University of Minnesota is committed to providing equitable access to learning opportunities for all students. Disability Services (DS) is the campus office that collaborates with students who have disabilities to provide and/or arrange reasonable accommodations.

If you have, or think you may have, a disability (e.g., mental health, attentional, learning, chronic health, sensory, or physical), please contact DS at 612-626-1333 to arrange a confidential discussion regarding equitable access and reasonable accommodations.

If you are registered with DS and have a current letter requesting reasonable accommodations, please contact your instructor as early in the semester as possible to discuss how the accommodations will be applied in the course.

For more information, please see the DS website, <https://diversity.umn.edu/disability/>.

### **Mental Health and Stress Management**

As a student you may experience a range of issues that can cause barriers to learning, such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished academic performance and may reduce your ability to participate in daily activities. University of Minnesota services are available to assist you. You can learn more about the broad range of confidential mental health services available on campus via the Student Mental Health Website: <http://www.mentalhealth.umn.edu>.

### **The Office of Student Affairs at the University of Minnesota**

The Office for Student Affairs provides services, programs, and facilities that advance student success, inspire students to make life-long positive contributions to society, promote an inclusive environment, and enrich the University of Minnesota community.

Units within the Office for Student Affairs include, the Aurora Center for Advocacy & Education, Boynton Health Service, Central Career Initiatives (CCE, CDes, CFANS), Leadership Education and Development – Undergraduate Programs (LEAD-UP), the Office for Fraternity and Sorority Life, the Office for Student Conduct and Academic Integrity, the Office for Student Engagement, the Parent Program, Recreational Sports, Student and Community Relations, the Student Conflict Resolution Center, the Student Parent HELP Center, Student Unions & Activities, University Counseling & Consulting Services, and University Student Legal Service.

For more information, please see the Office of Student Affairs at <http://www.osa.umn.edu/index.html>.

### **Academic Freedom and Responsibility, for courses that involve students in research**

Academic freedom is a cornerstone of the University. Within the scope and content of the course as defined by the instructor, it includes the freedom to discuss relevant matters in the classroom and conduct relevant research. Along with this freedom comes responsibility. Students are encouraged to develop the capacity for critical judgment and to engage in a sustained and independent search for truth. Students are free to take reasoned exception to the views offered in any course of study and to reserve judgment about matters of opinion, but they are responsible for learning the content of any course of study for which they are enrolled.\* When conducting research, pertinent institutional approvals must be obtained and the research must be consistent with University policies.

Reports of concerns about academic freedom are taken seriously, and there are individuals and offices available for help. Contact the instructor, the Department Chair, your adviser, the associate dean of the college, or the Vice Provost for Faculty and Academic Affairs in the Office of the Provost.

\* Language adapted from the American Association of University Professors "Joint Statement on Rights and Freedoms of Students".

Template 9/2014 + modifications from the July 2019 SPH syllabus update