TABLE 28.11 Latin Square Crossover Design—Apple Sales Example.

(a) Data (coded)

Pattern i	Store	Two-Week Period (j) 1	2	3
1	$m=1$	9 (B)	12 (C)	15 (A)
	$m=2$	4 (B)	12 (C)	9 (A)
2	$m=1$	12 (A)	14 (B)	3 (C)
	$m=2$	13 (A)	14 (B)	3 (C)
3	$m=1$	7 (C)	18 (A)	6 (B)
	$m=2$	5 (C)	20 (A)	4 (B)

(b) Analysis of Variance

Source of Variation	SS	df	MS
Patterns	.33	2	.17
Order positions	233.33	2	116.67
Displays	189.00	2	94.50
Stores (within patterns)	21.00	3	7.00
Error	20.33	8	2.54
Total	464.0	17	

Example

Table 28.11a contains data for a study of the effects of three different displays on the sale of apples, using a latin square crossover design. Six stores were used, with two assigned at random to each of the three treatment order patterns shown. Each display was kept for two weeks, and the observed variable was sales per 100 customers. Table 28.11b contains the analysis of variance. The sums of squares were obtained from a computer run.

To test for treatment (display) effects, we use:

$$F^* = \frac{MSTR}{MSRem} = \frac{94.5}{2.54} = 37.2$$

For $\alpha = .05$, we require $F(.95; 2, 8) = 4.46$. Since $F^* = 37.2 > 4.46$, we conclude that there are differential sales effects for the three displays. The P-value of the test is 0+. Tests for pattern effects, order position effects, and store effects were also carried out. They indicated that order position effects were present, but no pattern or store effects. Order position effects here are associated with the three time periods in which the displays were studied, and may reflect seasonal effects as well as the results of special events, such as unusually hot weather in one period. The comparison of the three treatment effects was then carried out in the usual fashion.

Use of Independent Latin Squares

If the order position effects are not approximately constant for all subjects (stores, etc.), a crossover design is not fully effective. It may then be preferable to place the subjects into homogeneous groups with respect to the order position effects and use independent latin

squares for each group. Suppose that four treatments are to be administered to eight subjects each, four males and four females, and that the experimenter expects the fatigue effect to be stronger for females than for males. The use of two independent latin squares, one for male subjects and the other for female subjects, may then be advisable.

Carryover Effects

If carryover effects from one treatment to another are anticipated, that is, if not only the order position but also the preceding treatment has an effect, these carryover effects may be balanced out by choosing a latin square in which every treatment follows every other treatment an equal number of times. For $r = 4$, an example of such a latin square is:

Subject	Period 1	2	3	4
1	A	B	D	C
2	B	C	A	D
3	C	D	B	A
4	D	A	C	B

Note that treatment A follows each of the other treatments once, and similarly for the other treatments. This design is appropriate when the carryover effects do not persist for more than one period.

When r is odd, the sequence balance can be obtained by using a pair of latin squares with the property that the treatment sequences in one square are reversed in the other square. Indeed, even when r is even, it is usually desirable to use a pair of such squares so that the degrees of freedom associated with $MSRem$ are reasonably large. Such a design is sometimes called a *latin square double crossover design*. This type of design retains the advantages of employing two blocking variables in a latin square, while enabling the experimenter also to balance and measure the carryover effects.

For the earlier apple display illustration in which three displays were studied in six stores, the two latin squares might be as shown in Table 28.12. The stores should first be placed into two homogeneous groups and these should then be assigned to the two latin squares.

TABLE 28.12 Illustration of a Latin Square Double Crossover Design.

Square	Store	Two-Week Period 1	2	3
1	1	A	B	C
	2	B	C	A
	3	C	A	B
2	4	A	C	B
	5	B	A	C
	6	C	B	A