Orthogonality and Un-correlation

As noted previously, for 3 models:

Model 1: $z_i = \mu + \alpha x_i + \varepsilon_i$

Model 2: $z_i = \mu + \beta y_i + \varepsilon_i$

Model 3: $z_i = \mu + \alpha x_i + \beta y_i + \varepsilon_i$,

If the vectors $(x_1, x_2, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are orthogonal [i.e. $\sum x_i y_i = 0$] then the model sums of squares in the ANOVA table are related as:

$$SS_{\text{Model 3}} = SS_{\text{Model 1}} + SS_{\text{Model 2}}$$

Orthogonality of $\vec{x}$ and $\vec{y}$ is not the same as un-correlated-ness of $\vec{x}$ and $\vec{y}$. If

$$n \sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right) = 0$$

then $\vec{x}$ and $\vec{y}$ are uncorrelated.

So the question is: If $\vec{x}$ and $\vec{y}$ are <u>uncorrelated</u>, does this also imply that

$$SS_{\text{Model 3}} = SS_{\text{Model 1}} + SS_{\text{Model 2}} \ ?$$

The answer turns out to be <u>yes</u>.

Here is the reasoning:

1. If $\vec{x}$ and $\vec{y}$ are <u>centered</u>, then
   uncorrelated $\Rightarrow$ orthogonal.

   Def. centered: $\bar{x} = 0$, $\bar{y} = 0$.

   → Prove this for yourself [easy].

2. You can easily obtain a centered vector ~~to~~ from an uncentered one by the following replacement

$$(x_1, x_2, \ldots, x_n) \rightarrow (x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x})$$

$$\vec{x} \quad \rightarrow \quad \vec{x}_{\text{centered}}$$

3. The regression of $\vec{z}$ on $\vec{x}$ and/or $\vec{y}$ is equivalent to the regression of $\vec{z}$ on $\vec{x}_{\text{centered}}$ and/or $\vec{y}_{\text{centered}}$.

4. The model sums of squares for the centered predictors $\vec{x}_{\text{centered}}$ and/or $\overline{y}_{\text{centered}}$ are the same as the model sums of squcre fo $\vec{x}$ and/or $\vec{y}$.