Non-Parametric Tests Comparing Many Groups

PROC NPAR1WAY has several non-parametric tests for comparing the means (or medians) of more than two groups. All tests are based on each observation’s rank, and are computed as follows:

a. Rank all observations together from smallest to largest (not within groups). Tied observations get assigned the average rank. Denote the ranks by $R_k$, $k=1,\ldots,N$.

b. Compute a score based on the ranks.
Test #1: Median (Brown-Mood) test

\[
a_k = \begin{cases} 
1 & \text{if } R_k > \text{median}\{R_k, k = 1, \ldots N\} \\
0 & \text{if } R_k \leq \text{median}\{R_k, k = 1, \ldots N\}
\end{cases}
\]

Test #2: Wilcoxon (Kruskal-Wallis) test

\[
a_k = R_k
\]

Test #3: Van der Waerden test

\[
a_k = \Phi^{-1}\left(\frac{R_k}{N + 1}\right)
\]
c. Compute the sum of scores within group.

\[ T_i = \sum_{k=1}^{N} \delta_k a_k \quad \text{where} \quad \delta_k = \begin{cases} 1 & \text{obs. } k \text{ in group } i \\ 0 & \text{otherwise} \end{cases} \]

d. Compute a test statistic.

\[ C = \frac{\sum_{i=1}^{t} (T_i - \hat{E}_0[T_i])^2 / r_i}{s^2} \]

where \( \hat{E}_0[T_i] = r_i \bar{a} \)

\[ s^2 = \frac{1}{N - 1} \sum_{k=1}^{N} (a_k - \bar{a})^2 \]
e. Reject H₀: no group differences at level $\alpha$ if

$$C > \chi^2_{\alpha,t-1}$$

Coding: SAS – PROC NPAR1WAY

S-Plus & R – kruskal.test()
Notes:

- The median test has very low power for small samples. Otherwise it has good power, especially if the true distribution of the outcome is heavy-tailed. It tests medians:

  \[ H_0: \text{median}_1 = \text{median}_2 = \ldots = \text{median}_t \]

  and makes no assumptions about the shapes of the distributions for each group. If the true distribution for each group is symmetric, then this is equivalent to a test of means.
• The Kruskal-Wallis test has good power for skewed or heavy-tailed distributions, but only if each group’s distribution has approximately the same shape. It tests:

\[ H_0: \text{distribution the same for all groups} \]

\[ H_a: \text{same shaped distribution for all groups but different medians} \]

• The Van der Waerden test has the best power when the data are close to normally distributed. It tests the same hypothesis as the Kruskal-Wallis test.