"Latent growth" curve in Chpt 10 Kline presented differently

random intercep, random slope

$$y_{it} = \beta_{0i} + \beta_{1i} t + \varepsilon_{it}$$

$i = 1 \ldots n$ # of individuals
$t = 1 \ldots T$ # of time points

$$\begin{pmatrix} \beta_{0i} \\ \beta_{1i} \end{pmatrix} \sim N\left(\begin{pmatrix} b_0 \\ b_1 \end{pmatrix}, \begin{pmatrix} \sigma_0 & \sigma_{01} \\ \sigma_{01} & \sigma_1 \end{pmatrix} \right)$$

rewrite like factor analysis model

if T=4 at times 1 2 3 4

$$\begin{pmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \beta_{0i} + \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \beta_{1i} + \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \end{pmatrix}$$

$$\begin{pmatrix} \underset{\sim}{y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \beta_{0i} \\ \beta_{1i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \end{pmatrix}$$

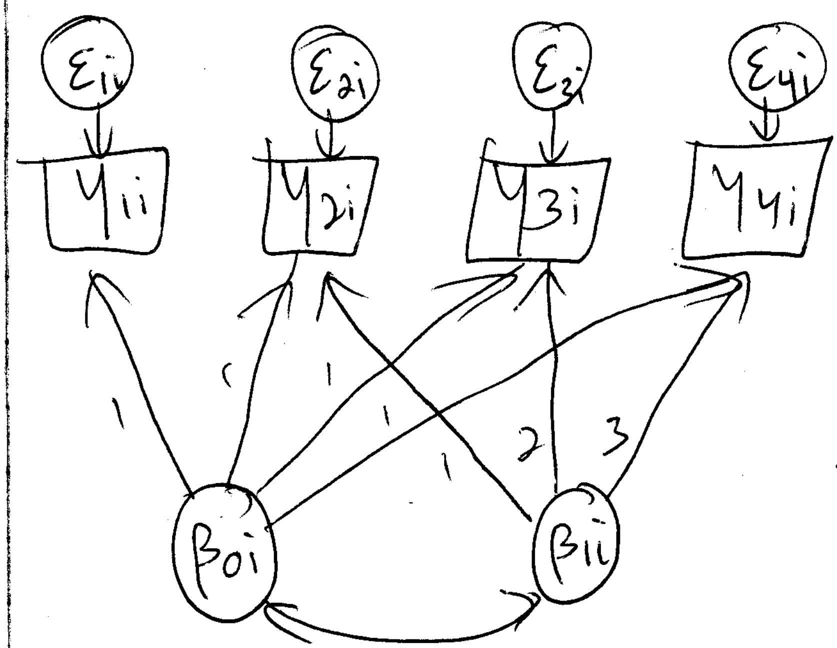
$$\underset{\sim}{y} = \underset{\sim}{\Lambda} \, \underset{\sim}{f} + \underset{\sim}{\varepsilon}$$

$\underset{\sim}{\Lambda}$ fixed.

Similar/same model

Different names

- Proc mixed model
- Random coefficient model
- Hierarchical Linear model
- cluster model
- latent growth curve
- Multi level model

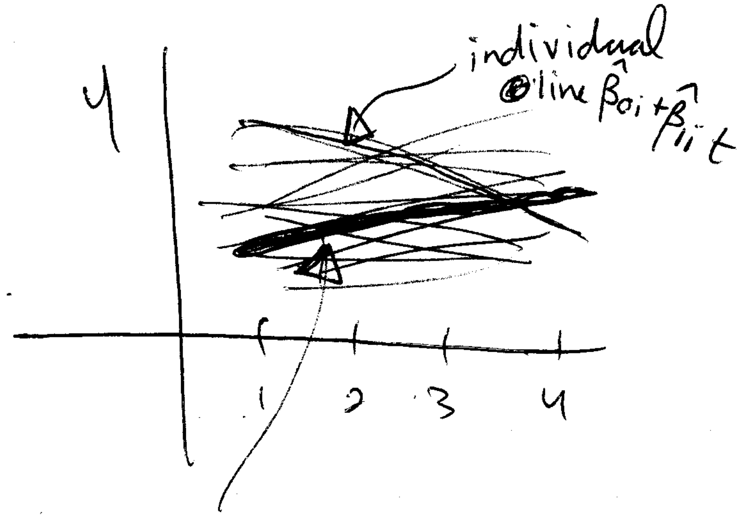


In Mplus
β_{0i} is called "i"
β_{1i} is called "s"

Notice no arrow drawn from β_{1i} to y_1 because fixed to zero

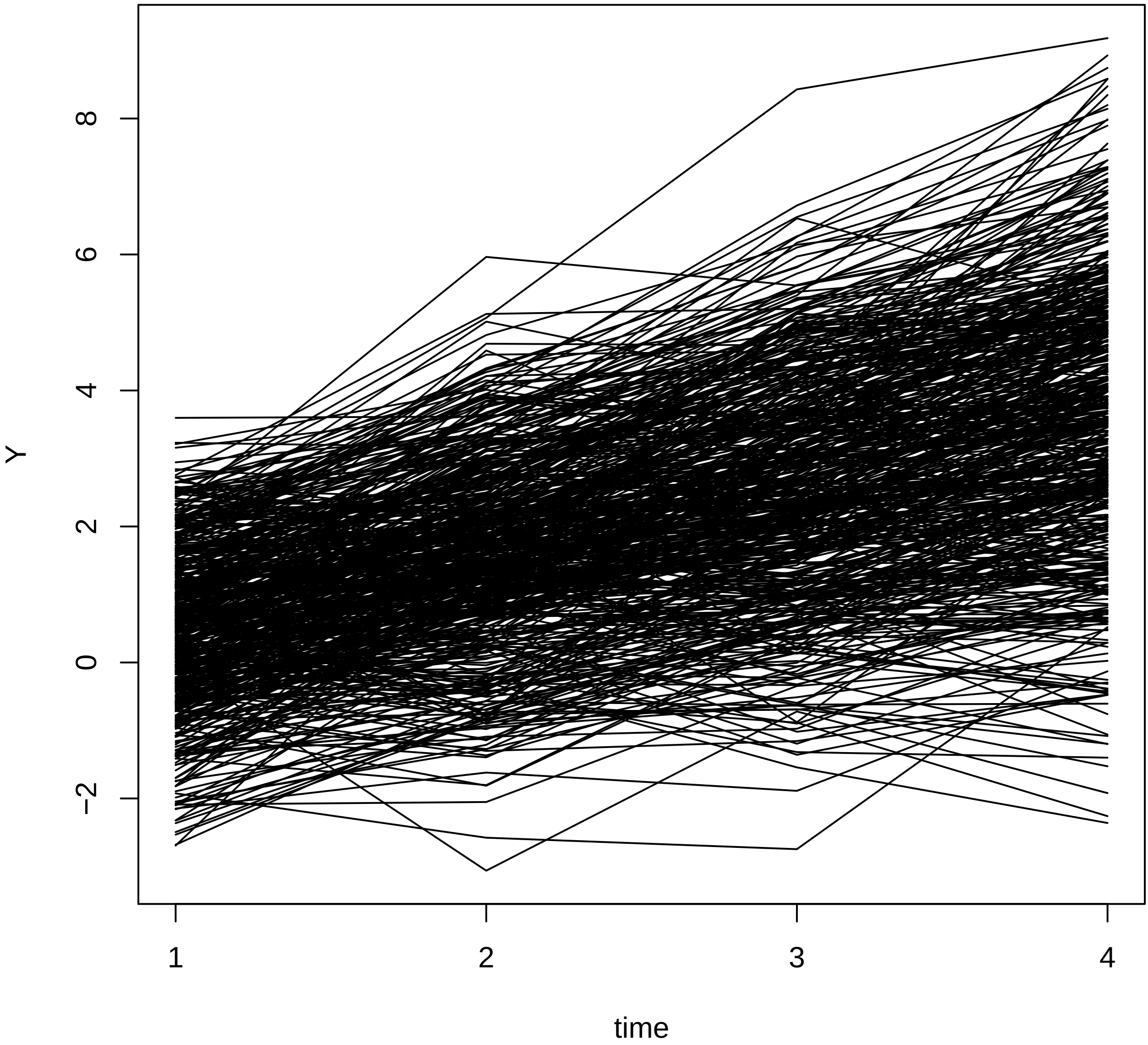
parameters of interest

$$\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} \text{ and } \begin{pmatrix} \sigma_0 & \\ \sigma_{12} & \sigma_1 \end{pmatrix}$$



overall mean: $b_0 + b_1 t$

Variability of intercepts
Variability of slopes.



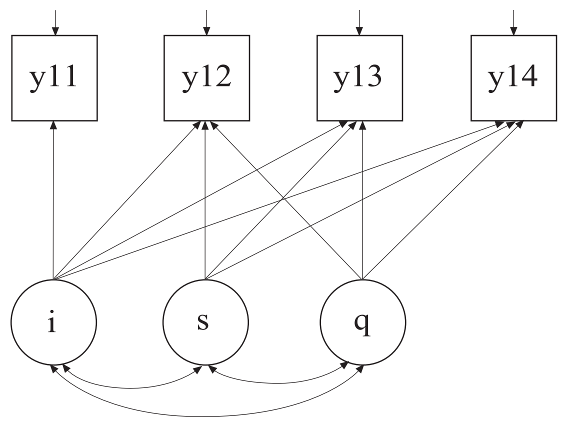
Y
8
6
4
2
0
−2
1
2
3
4
time

```
TITLE:    this is an example of a linear growth
          model for a continuous outcome
DATA:     FILE IS ex6.1.dat;
VARIABLE: NAMES ARE y11-y14;
MODEL:    i s | y11@0 y12@1 y13@2 y14@3;
Output: sampstat standardized;

SAMPLE STATISTICS


     SAMPLE STATISTICS


           Means
              Y11           Y12           Y13           Y14
              ________      ________      ________      ________
      1         0.514         1.566         2.568         3.601


           Covariances
              Y11           Y12           Y13           Y14
              ________      ________      ________      ________
 Y11            1.452
 Y12            1.133         1.978
 Y13            1.226         1.870         2.937
 Y14            1.392         2.166         3.019         4.307


           Correlations
              Y11           Y12           Y13           Y14
              ________      ________      ________      ________
 Y11            1.000
 Y12            0.668         1.000
 Y13            0.594         0.776         1.000
 Y14            0.557         0.742         0.849         1.000


THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL RESULTS

                   Estimates     S.E.  Est./S.E.    Std     StdYX

 I         |
    Y11                1.000    0.000      0.000    0.994    0.822
    Y12                1.000    0.000      0.000    0.994    0.710
    Y13                1.000    0.000      0.000    0.994    0.585
```

```
    Y14                1.000      0.000      0.000      0.994      0.477

 S        |
    Y11                0.000      0.000      0.000      0.000      0.000
    Y12                1.000      0.000      0.000      0.473      0.338
    Y13                2.000      0.000      0.000      0.946      0.557
    Y14                3.000      0.000      0.000      1.419      0.681

 S        WITH
    I                  0.133      0.033      4.057      0.283      0.283

 Means
    I                  0.523      0.051     10.153      0.526      0.526
    S                  1.026      0.025     40.268      2.170      2.170

 Intercepts
    Y11                0.000      0.000      0.000      0.000      0.000
    Y12                0.000      0.000      0.000      0.000      0.000
    Y13                0.000      0.000      0.000      0.000      0.000
    Y14                0.000      0.000      0.000      0.000      0.000

 Variances
    I                  0.989      0.089     11.097      1.000      1.000
    S                  0.224      0.023      9.891      1.000      1.000

 Residual Variances
    Y11                0.475      0.059      7.989      0.475      0.324
    Y12                0.482      0.040     11.994      0.482      0.246
    Y13                0.473      0.047     10.007      0.473      0.164
    Y14                0.545      0.084      6.471      0.545      0.125


R-SQUARE

    Observed
    Variable  R-Square

    Y11          0.676
    Y12          0.754
    Y13          0.836
    Y14          0.875
```

EXAMPLE 6.9: QUADRATIC GROWTH MODEL FOR A CONTINUOUS OUTCOME

```
TITLE:     this is an example of a quadratic growth
           model for a continuous outcome
DATA:      FILE IS ex6.9.dat;
VARIABLE:  NAMES ARE y11-y14 x1 x2 x31-x34;
           USEVARIABLES ARE y11-y14;
MODEL:     i s q | y11@0 y12@1 y13@2 y14@3;
```

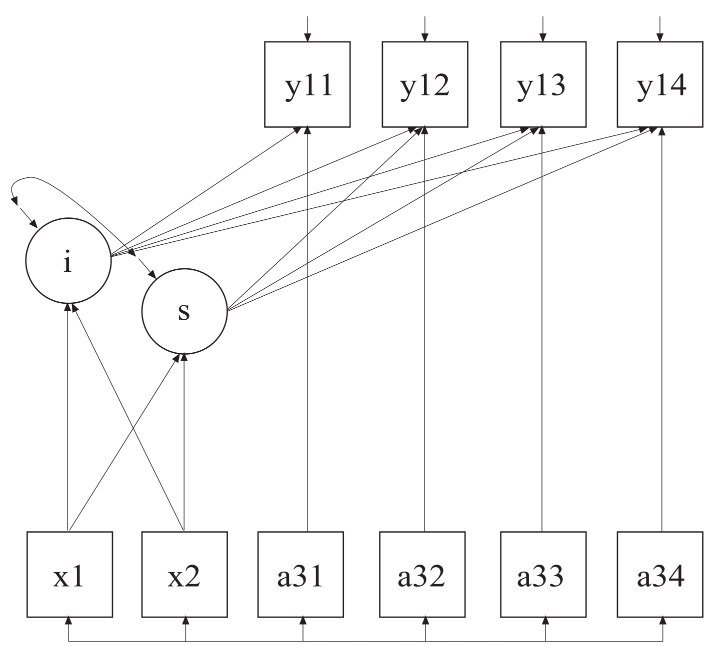


The difference between this example and Example 6.1 is that the quadratic growth model shown in the picture above is estimated. A quadratic growth model requires three random effects: an intercept factor (i), a linear slope factor (s), and a quadratic slope factor (q). The | symbol is used to name and define the intercept and slope factors in the growth model. The names i, s, and q on the left-hand side of the | symbol are the names of the intercept, linear slope, and quadratic slope factors, respectively. In the example above, the linear slope factor has equidistant time scores of 0, 1, 2, and 3. The time scores for the quadratic slope factor are the squared values of the linear time scores. These time scores are automatically computed by the program.

In the parameterization of the growth model shown here, the intercepts of the outcome variable at the four time points are fixed to zero as the default. The means and variances of the three growth factors are estimated as the default, and the three growth factors are correlated as the default because they are independent (exogenous) variables. The default estimator for this type of analysis is maximum likelihood. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 6.1.

EXAMPLE 6.10: LINEAR GROWTH MODEL FOR A CONTINUOUS OUTCOME WITH TIME-INVARIANT AND TIME-VARYING COVARIATES

```
TITLE:     this is an example of a linear growth
           model for a continuous outcome with time-
           invariant and time-varying covariates
DATA:      FILE IS ex6.10.dat;
VARIABLE:  NAMES ARE y11-y14 x1 x2 a31-a34;
MODEL:     i s | y11@0 y12@1 y13@2 y14@3;
           i s ON x1 x2;
           y11 ON a31;
           y12 ON a32;
           y13 ON a33;
           y14 ON a34;
```

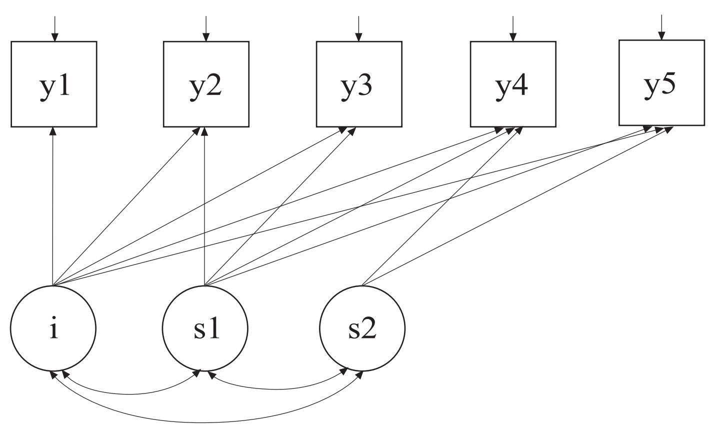


The difference between this example and Example 6.1 is that time-invariant and time-varying covariates as shown in the picture above are included in the model.

The first ON statement describes the regressions of the two growth factors on the time-invariant covariates x1 and x2. The next four ON statements describe the regressions of the outcome variable on the time-varying covariates a31, a32, a33, and a34 at each of the four time points. It is not necessary to refer to the means, variances, and covariances among the covariates in the MODEL command. They are automatically estimated as the sample values. The default estimator for this type of analysis is maximum likelihood. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 6.1.

EXAMPLE 6.11: PIECEWISE GROWTH MODEL FOR A CONTINUOUS OUTCOME

```
TITLE:     this is an example of a piecewise growth
           model for a continuous outcome
DATA:      FILE IS ex6.11.dat;
VARIABLE:  NAMES ARE y1-y5;
MODEL:     i s1 | y1@0 y2@1 y3@2 y4@2 y5@2;
           i s2 | y1@0 y2@0 y3@0 y4@1 y5@2;
```

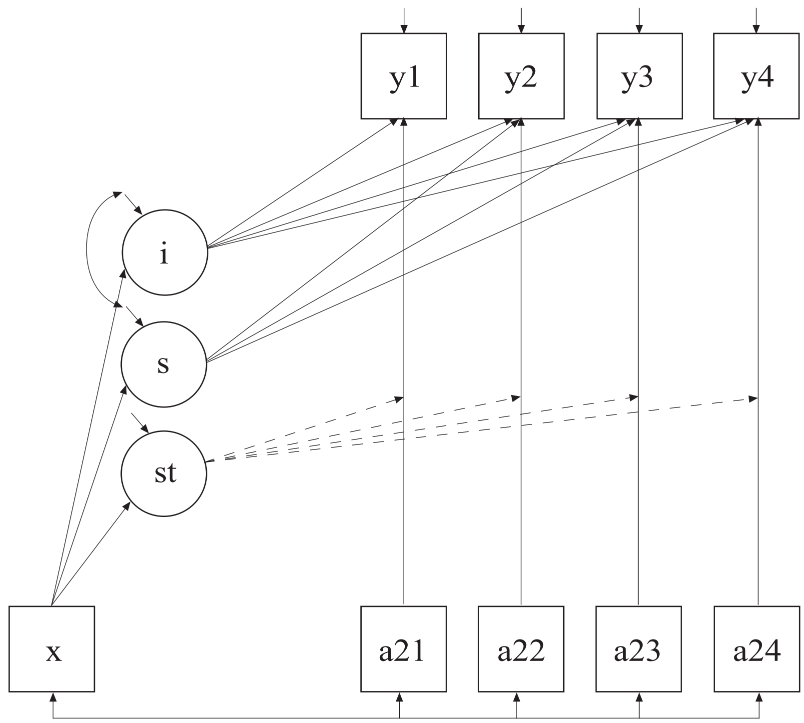


In this example, the piecewise growth model shown in the picture above is estimated. In a piecewise growth model, different phases of development are captured by more than one slope growth factor. The first | statement specifies a linear growth model for the first phase of development which includes the first three time points. The second | statement specifies a linear growth model for the second phase of development which includes the last three time points. Note that there is one intercept growth factor i. It must be named in the specification of both growth models when using the | symbol.

In the parameterization of the growth models shown here, the intercepts of the outcome variable at the five time points are fixed to zero as the default. The means and variances of the three growth factors are estimated as the default, and the three growth factors are correlated as the default because they are independent (exogenous) variables. The default estimator for this type of analysis is maximum likelihood. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 6.1.

EXAMPLE 6.12: GROWTH MODEL WITH INDIVIDUALLY-VARYING TIMES OF OBSERVATION AND A RANDOM SLOPE FOR TIME-VARYING COVARIATES FOR A CONTINUOUS OUTCOME

```
TITLE:     this is an example of a growth model with
           individually-varying times of observation
           and a random slope for time-varying
           covariates for a continuous outcome
DATA:      FILE IS ex6.12.dat;
VARIABLE:  NAMES ARE y1-y4 x a11-a14 a21-a24;
           TSCORES = a11-a14;
ANALYSIS:  TYPE = RANDOM;
MODEL:     i s | y1-y4 AT a11-a14;
           st | y1 ON a21;
           st | y2 ON a22;
           st | y3 ON a23;
           st | y4 ON a24;
           i s st ON x;
```



In this example, the growth model with individually-varying times of observation, a time-invariant covariate, and time-varying covariates with random slopes shown in the picture above is estimated.

The TSCORES option is used to identify the variables in the data set that contain information about individually-varying times of observation for the outcomes. The TYPE option is used to describe the type of analysis that is to be performed. By selecting RANDOM, a growth model with random slopes will be estimated.

The | symbol is used to name and define the random effect variables in the model. The names on the left-hand side of the | symbol name the random effect variables. In the first | statement, the AT option is used on the right-hand side of the | symbol to define a growth model with individually-varying times of observation for the outcome variable. Two growth factors are used in the model, a random intercept, i, and a random slope, s.

In the parameterization of the growth model shown here, the intercepts of the outcome variables are fixed to zero as the default. The residual variances of the outcome variables are free to be estimated as the default. The residual covariances of the outcome variables are fixed to zero as the default. The means, variances, and covariances of the intercept and slope growth factors are free as the default.

In the second, third, fourth, and fifth | statements, ON statements on the right-hand side of the | symbol identify the regressions that have a random slope, st. In the last ON statement, the random effect variables are regressed on the covariate x. The intercepts and residual variances of the random effect variables, i, s, and st, are free as the default. The residual covariance between i and s is estimated as the default. The residual covariances between st and i and s are fixed at zero as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The estimator option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 6.1.

EXAMPLE 6.13: GROWTH MODEL FOR TWO PARALLEL PROCESSES FOR CONTINUOUS OUTCOMES WITH REGRESSIONS AMONG THE RANDOM EFFECTS

```
TITLE:     this is an example of a growth model for
           two parallel processes for continuous
           outcomes with regressions among the random
           effects
DATA:      FILE IS ex6.13.dat;
VARIABLE:  NAMES ARE y11 y12 y13 y14 y21 y22 y23 y24;
MODEL:     i1 s1 | y11@0 y12@1 y13@2 y14@3;
           i2 s2 | y21@0 y22@1 y23@2 y24@3;
           s1 ON i2;
           s2 ON i1;
```