Understanding hypothesis testing comparing proportions.

[Example adapted from: Diane Essex-Sorlie, Medical Biostatistics and Epidemiology, page 218]

The association between the HLA-DR4 histocompatibility antigen and rheumatoid arthritis is investigated in randomly selected patients with and without rheumatoid arthritis. It is postulated that a greater proportion of patients with rheumatoid arthritis will be HLA-DR4 positive compared to healthy patients without rheumatoid arthritis. In a sample of 103 patients with rheumatoid arthritis, 76 are HLA-DR4 positive, compared to 27 of 51 healthy patients (people without rheumatoid arthritis).

Use a hypothesis test to determine if the proportion of HLA-DR4 positive patients is greater for the patients with rheumatoid arthritis compared those who are healthy. 

\[ P(Z < 2.59) = 0.9952 \]

\[
\begin{align*}
\text{R.A.} & \quad \text{Healthy} \\
\hat{p}_1 & = \frac{76}{103} & \hat{p}_2 & = \frac{27}{51}
\end{align*}
\]

\[
\begin{align*}
\hat{p} & = \frac{\chi_1 + \chi_2}{n_1 + n_2} = \frac{76 + 27}{103 + 51} = 0.6688 \\
Z & = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{\frac{76}{103} - \frac{27}{51}}{\sqrt{0.6688(1-0.6688)(\frac{1}{103} + \frac{1}{51})}} \\
& = 2.59
\end{align*}
\]

\[ P_{value} = P(Z > 2.59) = 1 - 0.9952 = 0.0048 \]

\[ \text{Reject } H_0 \]