Data Set: Seed germination study from Crowder(1978)

The study concerns about the proportion of seeds that germinated on each of 21 plates arranged according to a 2 by 2 factorial layout by seed and type of root extract;

- \( r_i \) is number of germinated seeds in the \( i^{th} \) plate;
- \( n_i \) is the total number of seeds in the \( i^{th} \) plate.

<table>
<thead>
<tr>
<th>seed O. aegyptiaco</th>
<th>75</th>
<th>76</th>
<th>83</th>
<th>8</th>
<th>16</th>
<th>0.50</th>
<th>3</th>
<th>12</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bean</td>
<td>10</td>
<td>39</td>
<td>0.26</td>
<td>5</td>
<td>6</td>
<td>0.83</td>
<td>8</td>
<td>28</td>
<td>0.29</td>
</tr>
<tr>
<td>Cucumber</td>
<td>23</td>
<td>62</td>
<td>0.37</td>
<td>53</td>
<td>74</td>
<td>0.72</td>
<td>10</td>
<td>30</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Random Effect Logistic Regression

- Denote \( p_i \) by the probability of germination on the \( i^{th} \) plate;
- \( r_i \sim \text{Binomial}(p_i, n_i) \);
- \( \text{logit}(p_i) = \alpha_0 + \alpha_1 x_{1i} + \alpha_2 x_{2i} + \alpha_{12} x_{1i} x_{2i} + b_i \);
- \( b_i \sim \text{Normal}(0, \tau) \);
- \( \alpha_0, \alpha_1, \alpha_2, \alpha_{12}, \tau \) are given independent “noninformative prior”;

where \( X_{1i}, X_{2i} \) are the seed type and root extract of the \( i^{th} \) plate, respectively.