WinBUGS Example 1: Lip cancer

- Consider the areal data disease mapping model:

$$
\begin{aligned}
Y_i \mid \mu_i \;&\stackrel{ind}{\sim}\; Po\left(E_i\, e^{\mu_i}\right)\;, \quad \text{where} \\
Y_i \;&=\; \text{observed disease count,} \\
E_i \;&=\; \text{expected count (known), and} \\
\mu_i \;&=\; \mathbf{x}_i'\boldsymbol{\beta} + \theta_i + \phi_i
\end{aligned}
$$

WinBUGS Example 1: Lip cancer

- Consider the areal data disease mapping model:

$$
\begin{aligned}
Y_i \mid \mu_i &\stackrel{ind}{\sim} Po\left(E_i\, e^{\mu_i}\right)\ , \quad \text{where} \\
Y_i &= \text{observed disease count,} \\
E_i &= \text{expected count (known), and} \\
\mu_i &= \mathbf{x}_i'\boldsymbol{\beta} + \theta_i + \phi_i
\end{aligned}
$$

- Y_i informs directly only about $\xi_i \equiv \theta_i + \phi_i$

WinBUGS Example 1: Lip cancer

- Consider the areal data disease mapping model:

$$\begin{aligned} Y_i \mid \mu_i &\overset{ind}{\sim} Po\left(E_i\, e^{\mu_i}\right)\ , \quad \text{where} \\ Y_i &= \text{observed disease count,} \\ E_i &= \text{expected count (known), and} \\ \mu_i &= \mathbf{x}_i'\boldsymbol{\beta} + \theta_i + \phi_i \end{aligned}$$

- Y_i informs directly only about $\xi_i \equiv \theta_i + \phi_i$
- The $\mathbf{x}_i$ are explanatory spatial covariates; typically $\boldsymbol{\beta}$ has a flat prior.

WinBUGS Example 1: Lip cancer

- Consider the areal data disease mapping model:

$$\begin{aligned} Y_i \mid \mu_i &\stackrel{ind}{\sim} Po\left(E_i\, e^{\mu_i}\right)\ , \quad \text{where} \\ Y_i &= \text{observed disease count,} \\ E_i &= \text{expected count (known), and} \\ \mu_i &= \mathbf{x}_i'\boldsymbol{\beta} + \theta_i + \phi_i \end{aligned}$$

- Y_i informs directly only about $\xi_i \equiv \theta_i + \phi_i$
- The $\mathbf{x}_i$ are explanatory spatial covariates; typically $\boldsymbol{\beta}$ has a flat prior.
- The θ_i capture heterogeneity among the regions via

$$\theta_i \stackrel{iid}{\sim} N(0\,,\ 1/\tau_h)\ ,$$

WinBUGS Example 1: Lip cancer

- and the ϕ_i capture regional clustering via a conditionally autoregressive (CAR) prior,

$$\phi_i \mid \phi_{j \neq i} \sim N(\bar{\phi}_i \, , \, 1/(\tau_c m_i)) \, ,$$

  where $\bar{\phi}_i = m_i^{-1} \Sigma_{j \in \partial_i} \, \phi_j$, ∂_i is the set of “neighbors” of region i, and m_i is the number of these neighbors

WinBUGS Example 1: Lip cancer

- and the ϕ_i capture regional clustering via a conditionally autoregressive (CAR) prior,

$$\phi_i \mid \phi_{j \neq i} \sim N(\bar{\phi}_i \, , \, 1/(\tau_c m_i)) \, ,$$

  where $\bar{\phi}_i = m_i^{-1} \Sigma_{j \in \partial_i} \, \phi_j$, ∂_i is the set of "neighbors" of region i, and m_i is the number of these neighbors

- The CAR prior is translation invariant, so typically we insist $\Sigma_{i=1}^{I} \phi_i = 0$ (imposed numerically after each MCMC iteration).

WinBUGS Example 1: Lip cancer

- and the ϕ_i capture regional clustering via a conditionally autoregressive (CAR) prior,

$$\phi_i \mid \phi_{j \neq i} \sim N(\bar{\phi}_i \, , \, 1/(\tau_c m_i)) \; ,$$

  where $\bar{\phi}_i = m_i^{-1} \Sigma_{j \in \partial_i} \, \phi_j$, ∂_i is the set of "neighbors" of region i, and m_i is the number of these neighbors

- The CAR prior is translation invariant, so typically we insist $\Sigma_{i=1}^{I} \phi_i = 0$ (imposed numerically after each MCMC iteration).

- Making the reparametrization from $(\boldsymbol{\theta}, \boldsymbol{\phi})$ to $(\boldsymbol{\theta}, \boldsymbol{\xi})$, we have the joint posterior

$$p(\boldsymbol{\theta}, \boldsymbol{\xi} | \mathbf{y}) \propto L(\boldsymbol{\xi}; \mathbf{y}) p(\boldsymbol{\theta}) p(\boldsymbol{\xi} - \boldsymbol{\theta}).$$

WinBUGS Example 1: Lip cancer

- This means that

$$p(\theta_i \mid \theta_{j\neq i}, \boldsymbol{\xi}, \mathbf{y}) \propto p(\theta_i)\, p(\xi_i - \theta_i \mid \{\xi_j - \theta_j\}_{j\neq i}) \ .$$

Since this distribution is free of the data $\mathbf{y}$, the θ_i are Bayesianly unidentified (and so are the ϕ_i).

WinBUGS Example 1: Lip cancer

- This means that

$$p(\theta_i \mid \theta_{j\neq i}, \boldsymbol{\xi}, \mathbf{y}) \propto p(\theta_i)\, p(\xi_i - \theta_i \mid \{\xi_j - \theta_j\}_{j\neq i}) \ .$$

  Since this distribution is free of the data $\mathbf{y}$, the θ_i are Bayesianly unidentified (and so are the ϕ_i).

- BUT: this does not preclude Bayesian learning about θ_i; this would instead require

$$p(\theta_i | \mathbf{y}) = p(\theta_i) \ .$$

  [Stronger condition: data have no impact on the marginal (not conditional) posterior.]

WinBUGS Example 1: Lip cancer

- Dilemma: Though unidentified, the θ_i and ϕ_i are interesting in their own right, as is

$$\alpha = \frac{sd(\boldsymbol{\phi})}{sd(\boldsymbol{\theta}) + sd(\boldsymbol{\phi})} \,,$$

  where $sd(\cdot)$ is the empirical marginal standard deviation. Are there vague but proper prior values τ_h and τ_c that

WinBUGS Example 1: Lip cancer

- Dilemma: Though unidentified, the θ_i and ϕ_i are interesting in their own right, as is

$$\alpha = \frac{sd(\boldsymbol{\phi})}{sd(\boldsymbol{\theta}) + sd(\boldsymbol{\phi})} \ ,$$

  where $sd(\cdot)$ is the empirical marginal standard deviation. Are there vague but proper prior values τ_h and τ_c that
  - lead to acceptable convergence behavior, but

WinBUGS Example 1: Lip cancer

- Dilemma: Though unidentified, the θ_i and ϕ_i are interesting in their own right, as is

$$\alpha = \frac{sd(\boldsymbol{\phi})}{sd(\boldsymbol{\theta}) + sd(\boldsymbol{\phi})} \ ,$$

  where $sd(\cdot)$ is the empirical marginal standard deviation. Are there vague but proper prior values τ_h and τ_c that
  - lead to acceptable convergence behavior, but
  - still allow Bayesian learning?

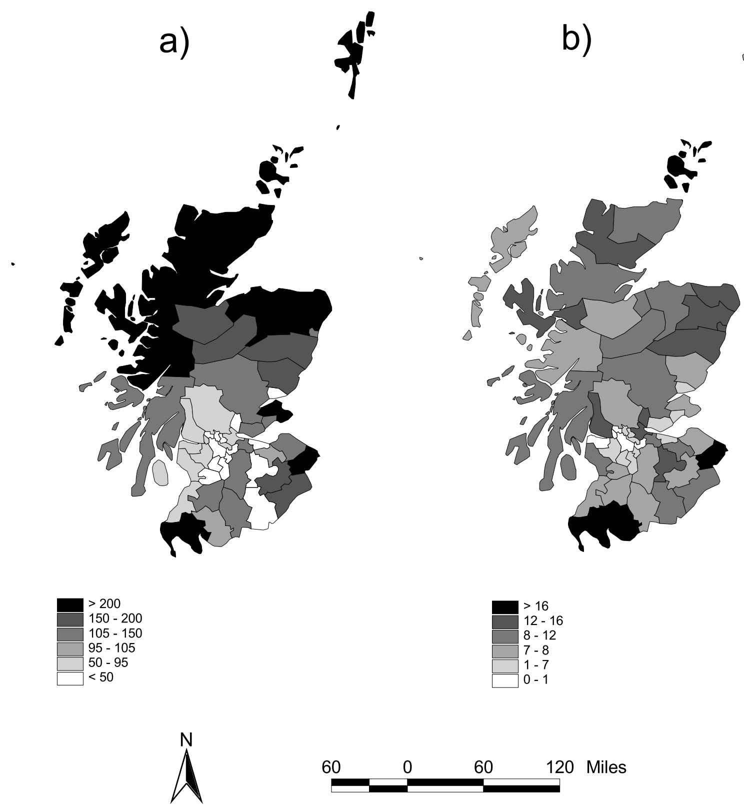
WinBUGS Example 1: Lip cancer

- Dilemma: Though unidentified, the θ_i and ϕ_i are interesting in their own right, as is

  $$\alpha = \frac{sd(\boldsymbol{\phi})}{sd(\boldsymbol{\theta}) + sd(\boldsymbol{\phi})} ,$$

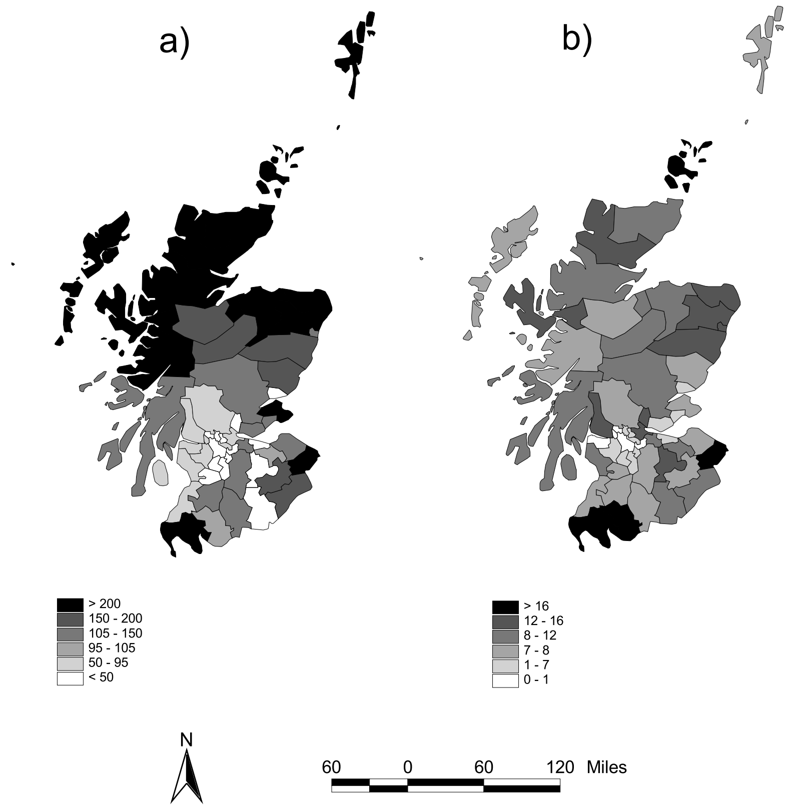
  where $sd(\cdot)$ is the empirical marginal standard deviation. Are there vague but proper prior values τ_h and τ_c that
  - lead to acceptable convergence behavior, but
  - still allow Bayesian learning?
- Tricky to specify a “fair” prior balance between heterogeneity and clustering (e.g., one for which $\alpha \approx 1/2$) since θ_i prior is specified marginally while the ϕ_i prior is specified conditionally!

WinBUGS Example 1: Lip cancer



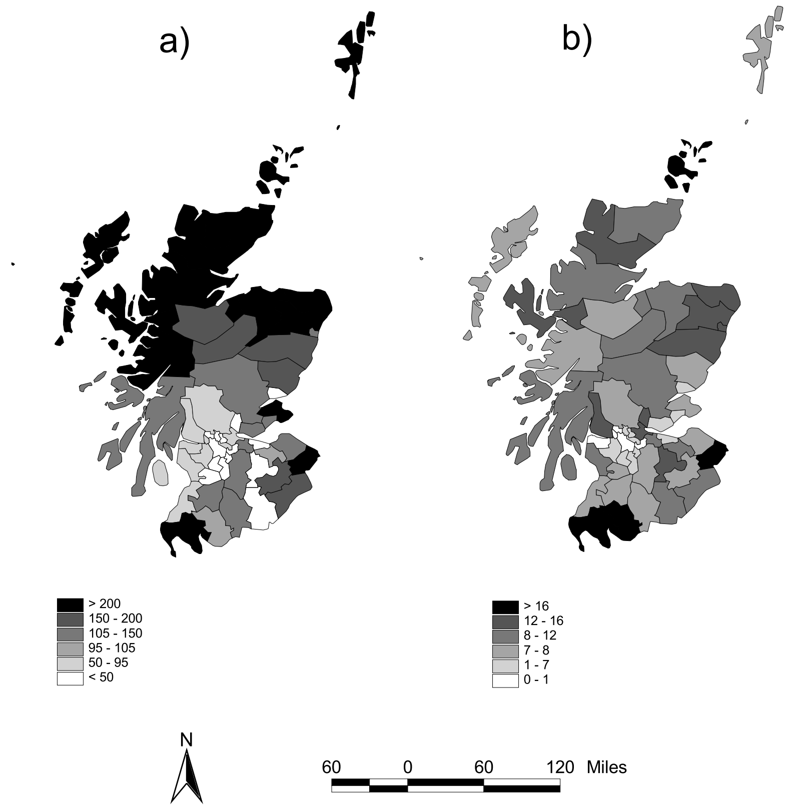
⋆ left panel: $100Y_i/E_i$ (SMR), where $Y_i =$ observed and $E_i =$ expected cases for $I = 56$ districts, 1975–1980

WinBUGS Example 1: Lip cancer



- ⋆ left panel: $100Y_i/E_i$ (SMR), where $Y_i =$ observed and $E_i =$ expected cases for $I = 56$ districts, 1975–1980
- ⋆ right panel: x_i, % of the population engaged in agriculture, fishing or forestry (AFF covariate)

WinBUGS Example 1: Lip cancer



- ⋆ left panel: $100Y_i/E_i$ (SMR), where Y_i = observed and E_i = expected cases for $I = 56$ districts, 1975–1980
- ⋆ right panel: x_i, % of the population engaged in agriculture, fishing or forestry (AFF covariate)
- ⋆ we also have: a variety of vague, proper, and arguably "fair" priors for τ_c and τ_h

WinBUGS Example 1: Lip cancer

For actual WinBUGS code, see:
http://www.biostat.umn.edu/∼brad/data/Lipsbrad.odc

Results:

- AFF covariate appears significantly different from 0 under all 3 priors, although convergence is very slow

WinBUGS Example 1: Lip cancer

For actual WinBUGS code, see:
http://www.biostat.umn.edu/∼brad/data/Lipsbrad.odc

Results:

- AFF covariate appears significantly different from 0 under all 3 priors, although convergence is very slow
- Excess variability in the data is mostly due to clustering ($E(\alpha|\mathbf{y}) > .50$), but the posterior distribution for α does not seem robust to changes in the prior.

WinBUGS Example 1: Lip cancer

For actual WinBUGS code, see:
http://www.biostat.umn.edu/∼brad/data/Lipsbrad.odc

Results:

- AFF covariate appears significantly different from 0 under all 3 priors, although convergence is very slow
- Excess variability in the data is mostly due to clustering ($E(\alpha|\mathbf{y}) > .50$), but the posterior distribution for α does not seem robust to changes in the prior.
- convergence for the ξ_i (reasonably well-identified) is rapid; convergence for the μ_i (not shown) is virtually immediate.

WinBUGS Example 1: Lip cancer

Posterior and MCMC convergence summaries:

	posterior for α			posterior for β		
priors for τ_c, τ_h	mean	sd	l1acf	mean	sd	l1acf
G(1.0, 1.0), G(3.2761, 1.81)	.57	.058	.80	.43	.17	.94
G(.1, .1), G(.32761, .181)	.65	.073	.89	.41	.14	.92
G(.1, .1), G(.001, .001)	.82	.10	.98	.38	.13	.91
	posterior for ξ_1			posterior for ξ_{56}		
priors for τ_c, τ_h	mean	sd	l1acf	mean	sd	l1acf
G(1.0, 1.0), G(3.2761, 1.81)	.92	.40	.33	–.96	.52	.12
G(.1, .1), G(.32761, .181)	.89	.36	.28	–.79	.41	.17
G(.1, .1), G(.001, .001)	.90	.34	.31	–.70	.35	.21

WinBUGS Example 2: Home prices

Here we illustrate a non-Gaussian model for point-referenced spatial data:

- Data: Observations are home values (based on recent real estate sales) at 50 locations in Baton Rouge, Louisiana, USA.

WinBUGS Example 2: Home prices

Here we illustrate a non-Gaussian model for point-referenced spatial data:

- Data: Observations are home values (based on recent real estate sales) at 50 locations in Baton Rouge, Louisiana, USA.
- The response $Y(\mathbf{s})$ is a binary variable, with

$$Y(\mathbf{s}) = \begin{cases} 1 & \text{if price is “high” (above the median)} \\ 0 & \text{if price is “low” (below the median)} \end{cases}$$

WinBUGS Example 2: Home prices

Here we illustrate a non-Gaussian model for point-referenced spatial data:

- Data: Observations are home values (based on recent real estate sales) at 50 locations in Baton Rouge, Louisiana, USA.
- The response $Y(\mathbf{s})$ is a binary variable, with

$$Y(\mathbf{s}) = \begin{cases} 1 & \text{if price is “high” (above the median)} \\ 0 & \text{if price is “low” (below the median)} \end{cases}$$

- Observed covariates include the house’s age and total living area

WinBUGS Example 2: Home prices

- We fit a generalized linear model where
  $Y(\mathbf{s}_i) \sim Bernoulli(p(\mathbf{s}_i)), \;\; logit(p(\mathbf{s}_i)) = \mathbf{x}^T(\mathbf{s}_i)\boldsymbol{\beta} + w(\mathbf{s}_i)$

WinBUGS Example 2: Home prices

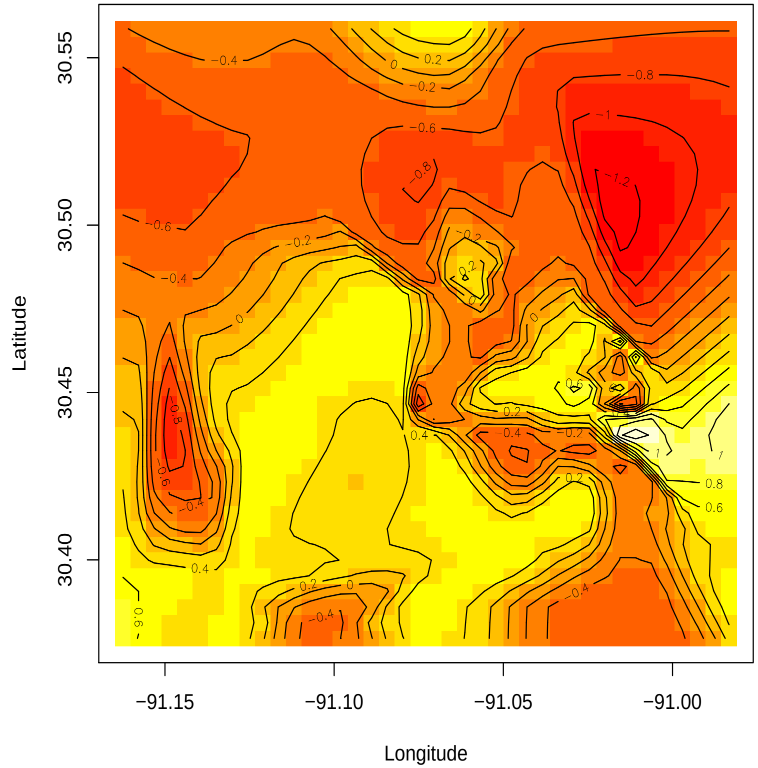
- We fit a generalized linear model where $Y(\mathbf{s}_i) \sim Bernoulli(p(\mathbf{s}_i)),\;\; logit(p(\mathbf{s}_i)) = \mathbf{x}^T(\mathbf{s}_i)\boldsymbol{\beta} + w(\mathbf{s}_i)$
- Assume vague priors for $\boldsymbol{\beta}$, a Uniform$(0, 10)$ prior for ϕ, and an Inverse Gamma$(0.1, 0.1)$ prior for σ^2.

WinBUGS Example 2: Home prices

- We fit a generalized linear model where $Y(\mathbf{s}_i) \sim Bernoulli(p(\mathbf{s}_i)), \;\; logit(p(\mathbf{s}_i)) = \mathbf{x}^T(\mathbf{s}_i)\boldsymbol{\beta} + w(\mathbf{s}_i)$
- Assume vague priors for $\boldsymbol{\beta}$, a Uniform$(0, 10)$ prior for ϕ, and an Inverse Gamma$(0.1, 0.1)$ prior for σ^2.
- The WinBUGS code and data for this example are at www.biostat.umn.edu/∼brad/data/BatonRougebinary.bug:

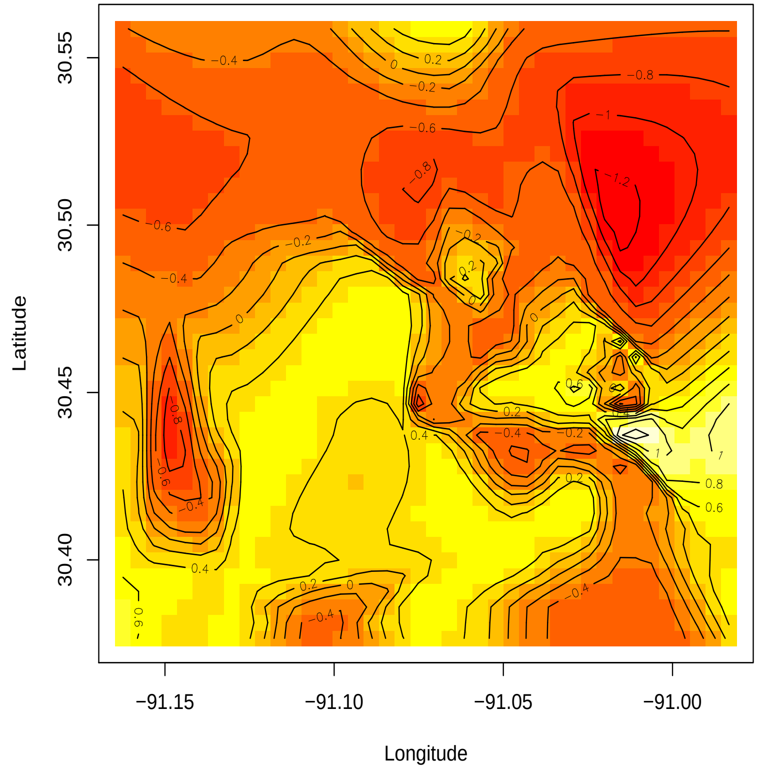
```
for (i in 1:N) {
      Y[i] ~ dbern(p[i])
      logit(p[i]) <- w[i]
      mu[i] <- beta[1]+beta[2]*LivingArea[i]/1000+beta[3]*Age[i] }
for (i in 1:3) beta[i] ~ dnorm(0.0,0.001)
w[1:N] ~ spatial.exp(mu[], x[], y[], spat.prec, phi, 1)
phi ~ dunif(0.1,10)
spat.prec ~ dgamma(0.1, 0.1)
sigmasq <- 1/spat.prec
```

WinBUGS Example 2: Home prices



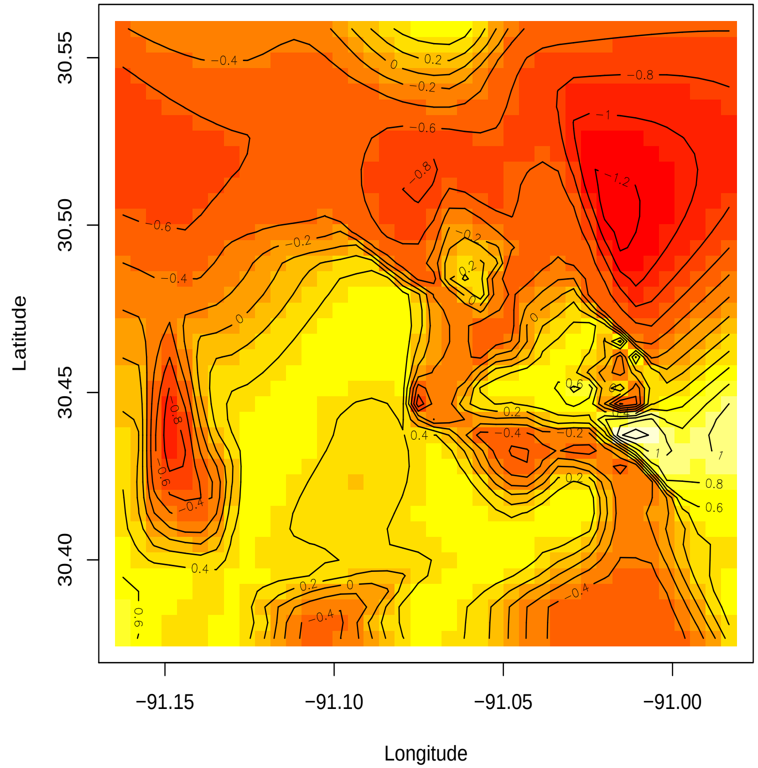
- Use `image` and `contour` on w_i posterior medians in `R`

WinBUGS Example 2: Home prices



- Use `image` and `contour` on w_i posterior medians in `R`
- negative residuals (i.e., lower prices) in the north; positive residuals (i.e., higher prices) in the south

WinBUGS Example 2: Home prices



- Use `image` and `contour` on w_i posterior medians in R
- negative residuals (i.e., lower prices) in the north; positive residuals (i.e., higher prices) in the south
- smooth flat stretches across the central parts; downward slopes toward the north and southeast.

WinBUGS Example 2: Home prices

Parameter estimates (posterior medians and upper and lower .025 points):

Parameter	50%	(2.5%, 97.5%)
β_1 (intercept)	–1.096	(–4.198, 0.4305)
β_2 (living area)	0.659	(–0.091, 2.254)
β_3 (age)	0.009615	(–0.8653, 0.7235)
ϕ	5.79	(1.236, 9.765)
σ^2	1.38	(0.1821, 6.889)

The covariate effects are generally uninteresting, though living area seems to have a marginally significant effect on price class.