Modeling the Golden Gate Bridge using Wireless Sensor Networks

Guilherme Rocha

Statistics Department
Indiana University, Bloomington

Joint Work with

Shamim Pakzad
CE, Lehigh Un.

Bin Yu
Stats, UC Berkeley

Acknowledgements:

David Culler	CS, UC Berkeley
James Demmel	CS, UC Berkeley
Greg Fenves	CE, UT Austin
Sukun Kim	Samsung

Outline

- **Motivation:**
  - Structural Health Monitoring
- **Physical model for vibration data:**
  - ModAl parameters
- **ModAl estimation from ambient vibrations**
  - Multivariate AR models (Full vs. Restricted)
- **Simulation Results**
  - Stabilization plots
- **Summary and Future work**

Structural Health Monitoring

- **National Bridge Inventory (DOT Report to Congress, 2004):**
  - Approx. 591,000 bridges in the U.S.
  - Approx. 81,000 (~14%) structurally deficient
  - Routine inspections by Federal Highway Adm. (FHWA):
    - Annually: ~71,000 bridges
    - Bi-annually: ~490,000 bridges
    - Every 4 years: ~28,000 bridges

- **Structural Health Monitoring Strategies:**
  - Direct damage detection
  - Indirect damage detection
    - <u>Detection of changes in dynamic properties of the structure</u>

Instrumentation
Accelerometer Characteristics

- **Sensor board:**
  - Two pairs of accelerometers:
    - Vertical and horizontal
    - Coarse and fine
  - One thermometer:
    - temperature for calibration

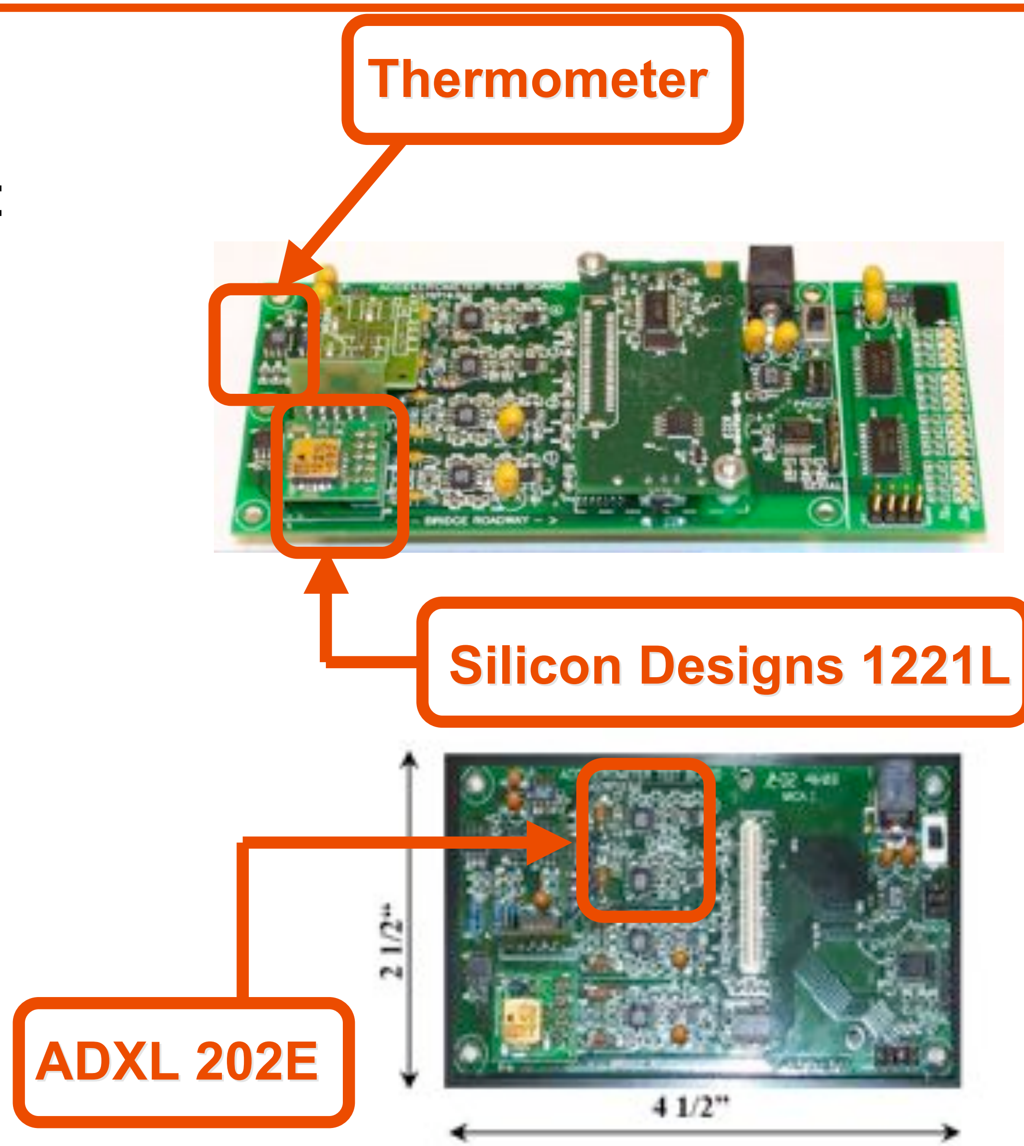
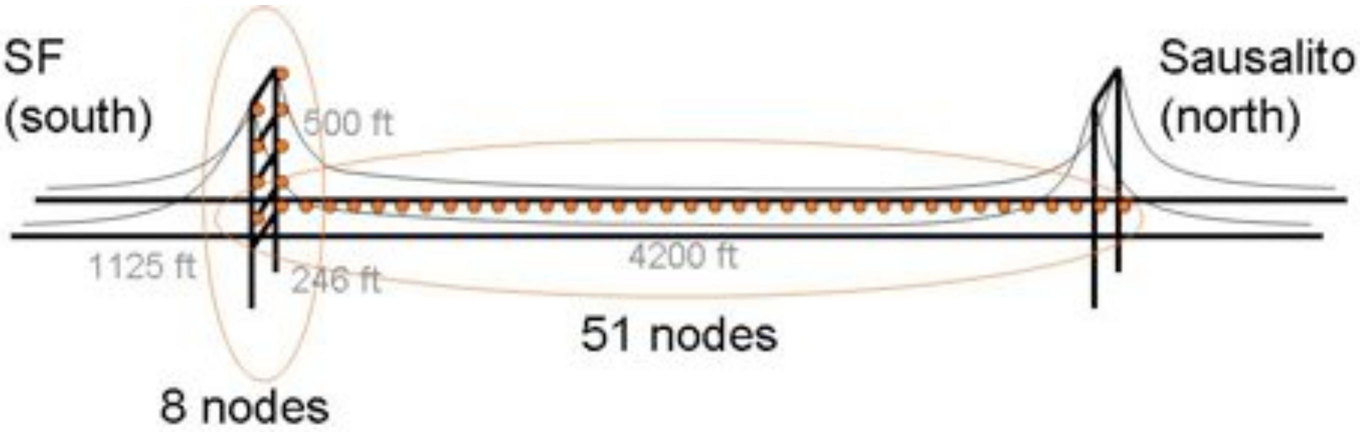


Figure adapted from Kim et. al. (2006)

Instrumentation:
Position of the sensors

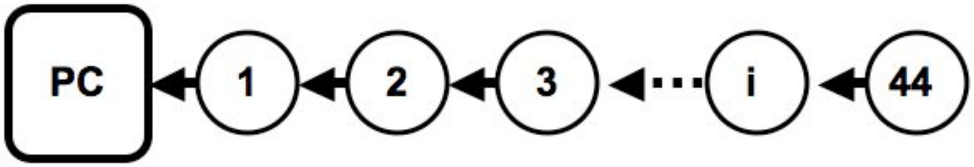


- **81 cables along west span of bridge**
- **8 sensors installed on south tower:**

Figure from Kim et. Al. (2006)

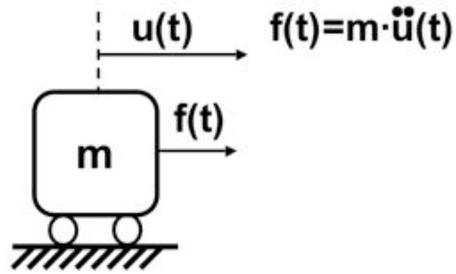
Data Transmission/Collection

- **Typical data generation rate:**
  - Uncompressed data rate 1KB/sec:
    - 2 bytes/sample (for both temperature and acceleration)
    - 5 samples/reading (1 temperature, 4 acceleration)
    - 100 readings/sec (sampling frequency: 100 Hz)
- **A typical run:**
  - 8 minutes @ 100Hz = 48,000 readings
  - 5 readings/sample @ 2 Bytes each = 10 Bytes/reading
  - Total transmitted volume: 480 Kbytes
- **Data rate of WSN:**
  - 1 hop away: ~1200 bytes/sec
  - 2 hops away: ~800 bytes/sec
  - 4 to 44 hops away: ~460 bytes/sec

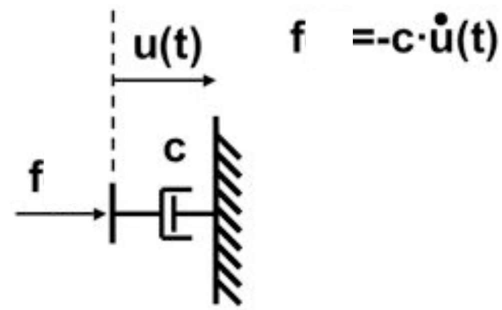


Vibrations 101
Lumped components

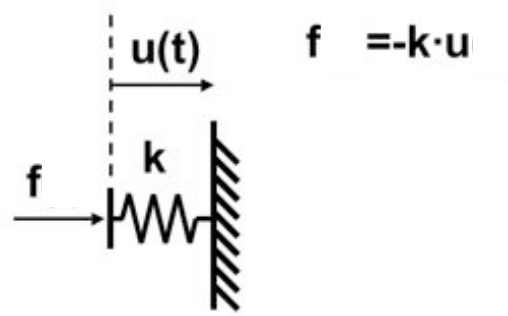
- **Masses:**
  - Model inertia
  - Force proportional to acceleration:
    - Newton's law



- **Dashpots:**
  - Model energy dissipation
  - Force proportional to speed:
    - Viscous friction

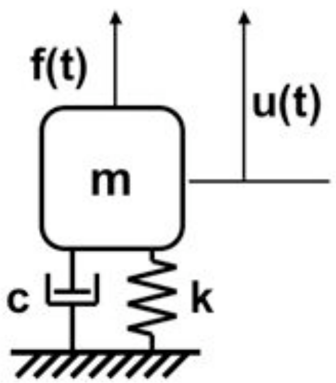


- **Springs:**
  - Model stiffness
  - Force proportional to displacement
    - Young's law

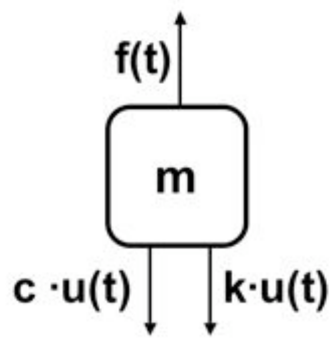


Vibrations 101
Natural frequency and damping ratio

Vibrating system:



Free body diagram and Newton's law:



$$m\ddot{u}(t) = f(t) - ku(t) - c\dot{u}(t)$$

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = f(t)$$

Assuming c<2mω, the homogeneous part of the ODE solution is:

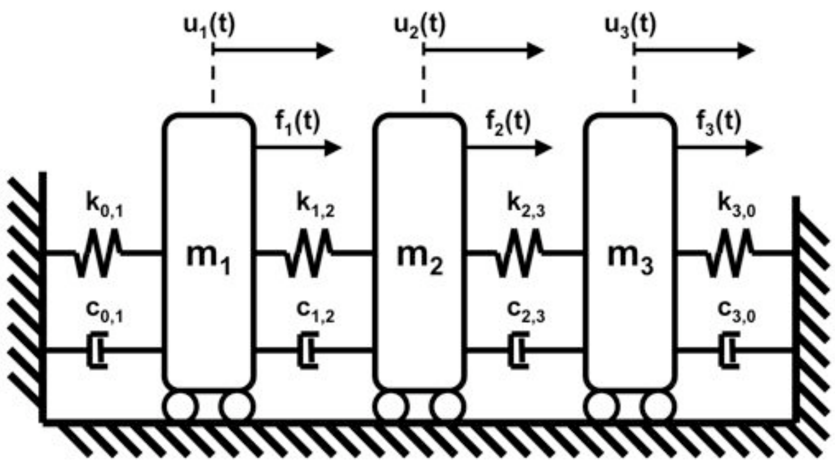
$$u(t) \quad = \quad \exp\left(-\zeta\omega t + i\frac{\omega t}{(1-\zeta^2)}\right)$$

$$\omega \quad := \quad \sqrt{\frac{k}{m}}$$

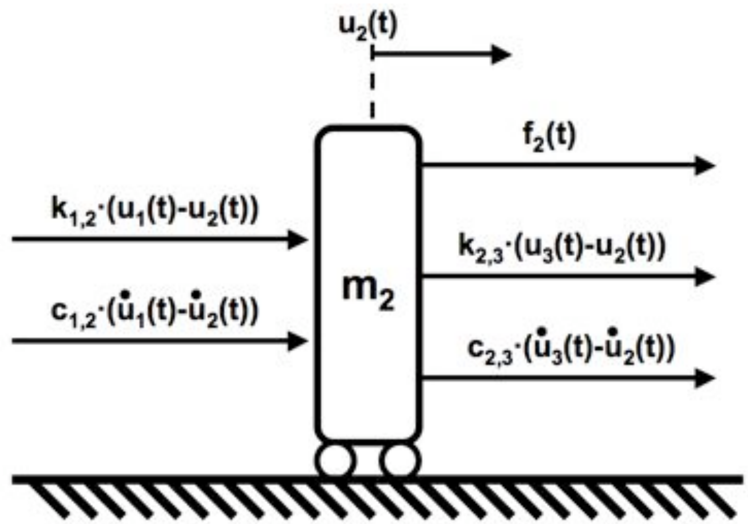
$$\zeta \quad := \quad \frac{c}{2m\omega}$$

Vibrations 101
A simple vibrating system

Diagram of vibrating system:



Free body diagram for mass i:



From free diagram of mass i:

$$m_i \ddot{u}_i + k_{0,i} u_i + c_{0,i} \dot{u}_i + \sum_{j \neq i} \left[k_{i,j}(u_j - u_i) + c_{i,j}(\dot{u}_j - \dot{u}_i) \right] = f_i$$

Collecting all equations, yields a multivariate system of coupled ODEs:

$$\mathbf{m}\ddot{\mathbf{u}}(t) + \mathbf{c}\dot{\mathbf{u}}(t) + \mathbf{k}\mathbf{u}(t) = \mathbf{f}(t)$$

Vibrations 101
Modes of vibration (1)

Undampened system: $\mathbf{m}\ddot{\mathbf{u}}(t) + \mathbf{k}\mathbf{u}(t) \quad = \quad \mathbf{f}(t)$

Gen. Eigenproblem: $\lambda_i \ \mathbf{m} \ \phi_i \quad = \quad \mathbf{k} \ \phi_i$

$$\begin{cases} \Phi & := & [\phi_1 \ \ \phi_2 \ \ \cdots \ \ \phi_p]\,, & \text{an orthogonal matrix,} \\ \mathbf{M} & := & \Phi^T \mathbf{m} \Phi, & \text{a diagonal matrix,} \\ \mathbf{K} & := & \Phi^T \mathbf{k} \Phi, & \text{a diagonal matrix,} \\ \mathbf{x} & := & \Phi^{-1} \mathbf{u}, & \text{a “state” variable.} \end{cases}$$

Leads to uncoupled system:

$$\mathbf{M}_{ii}\ddot{\mathbf{x}}_i(t) + \mathbf{K}_{ii}\mathbf{x}_i(t) \quad = \quad \phi_i^T \mathbf{f}(t), \qquad \text{for } i = 1, \ldots, p$$

Vibrations 101
Modes of vibration (2)

"Classical damping" assumption:

$$
\begin{aligned}
\text{Assume} \quad \mathbf{c} &= \alpha_0 \mathbf{m} + \alpha_1 \mathbf{k}, \\
\text{so} \quad \mathbf{C} &= \alpha_0 \mathbf{M} + \alpha_1 \mathbf{K}, \quad \text{is also diagonal.}
\end{aligned}
$$

Leads to p single degree of freedom systems:

$$\mathbf{M}_j \ddot{x}_j(t) + \mathbf{C}_j \dot{x}_j(t) + \mathbf{K}_j x_j(t) = \phi_j^T \mathbf{f}(t), \quad \text{for } j = 1, \ldots, p$$

Each ODE has homogeneous solution:

$$x_j(t) = \exp\left(-\zeta_j \omega_j t + i \frac{\omega_j t}{(1 - \zeta_j^2)}\right)$$

$$
\begin{aligned}
\omega_j &:= \sqrt{\frac{K_j}{M_j}} \\
\zeta_j &:= \frac{C_j}{2 M_j \omega_j}
\end{aligned}
$$

AR(q) models

- Use a multivariate AR model for acceleration data:

$$\ddot{\mathbf{u}}(k) = \sum_{j=1}^{q} \mathbf{B}_j \ddot{\mathbf{u}}(k-j) + \varepsilon(k) \qquad \mathbf{B}_j = \begin{bmatrix} \beta_{1,1,j} & \beta_{1,2,j} & \cdots & \beta_{1,p,j} \\ \beta_{2,1,j} & \beta_{2,2,j} & \cdots & \beta_{2,p,j} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{p,1,j} & \beta_{p,2,j} & \cdots & \beta_{p,p,j} \end{bmatrix}$$

- **Question:**

  How do we retrieve modes of vibration from AR model?

- **"Hand waving" answer:**

  Create "rotated variable" $x = \Phi^{-1}u$ such that

  x is well modeled by a "diagonal" AR model.

Banded models
From AR parameters to modes of vibration

For:

$$\ddot{\mathbf{u}}(k) = \sum_{j=1}^{q} \mathbf{B}_j \ddot{\mathbf{u}}(k-j) + \varepsilon(k)$$

define:

$$\ddot{\mathcal{U}}_q(k) = \left[\ddot{\mathbf{u}}(k)^T \;\; \ddot{\mathbf{u}}(k-1)^T \;\; \cdots \;\; \ddot{\mathbf{u}}(k-q+2)^T \;\; \ddot{\mathbf{u}}(k-q+1)^T \right]^T,$$

$$\mathcal{B}_q = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \cdots & \mathbf{B}_{q-1} & \mathbf{B}_q \\ \mathbf{I}_p & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_p & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_p & \mathbf{0} \end{bmatrix},$$

$$\ddot{\Upsilon}_q(k) = \left[\epsilon(k)^T \;\; \mathbf{0} \;\; \mathbf{0} \;\; \cdots \;\; \mathbf{0} \right]^T$$

to get:

$$\ddot{\mathcal{U}}_q(k) = \mathcal{B}_q \ddot{\mathcal{U}}_q(k-1) + \ddot{\Upsilon}_q(k)$$

Banded models
From AR parameters to modes of vibration

$$\ddot{\mathcal{U}}_q(k) \;=\; \mathcal{B}_q\ddot{\mathcal{U}}_q(k-1) + \ddot{\Upsilon}_q(k)$$

Let λ_j denote the j-th eigenvalue of companion matrix.

$$\mathcal{B}_q \;=\; \Phi_q \Lambda_q \Phi_q$$

$$\begin{aligned} \mu_j &= \log(\lambda_j) = -\omega_j\zeta_j \pm i\omega_j\sqrt{1-\zeta_j^2} \\ \omega_j &= \sqrt{\|\mu_j\|^2}, \text{ and} \\ \zeta_j &= \frac{\text{Real}(\mu_j)}{\omega_j}. \end{aligned}$$

$$\Phi_q = \begin{bmatrix} \phi_1\lambda_1^{q-1} & \phi_2\lambda_2^{q-1} & \cdots & \phi_{pq}\lambda_{pq}^{q-1} \\ \phi_1\lambda_1^{q-2} & \phi_2\lambda_2^{q-2} & \cdots & \phi_{pq}\lambda_{pq}^{q-2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1\lambda_1 & \phi_2\lambda_2 & \cdots & \phi_{pq}\lambda_{pq} \\ \phi_1 & \phi_2 & \cdots & \phi_{pq} \end{bmatrix}$$

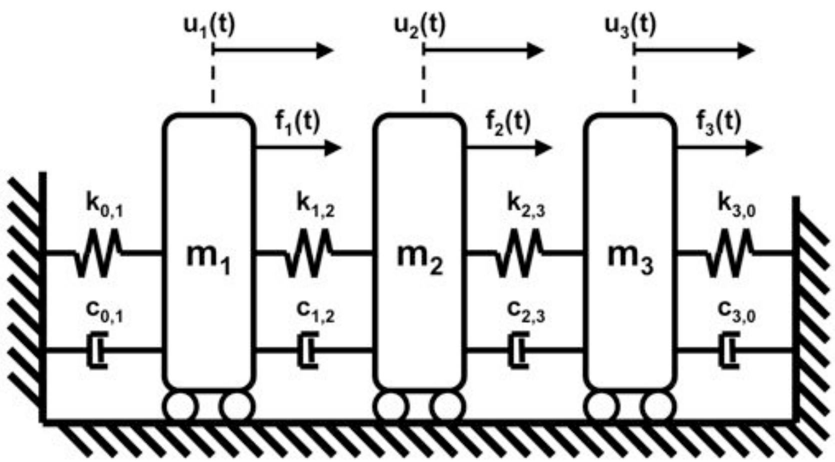
AR(q) models
Some issues

$$\ddot{\mathbf{u}}(k) = \sum_{j=1}^{q} \mathbf{B}_j \ddot{\mathbf{u}}(k-j) + \varepsilon(k) \qquad \mathbf{B}_j = \begin{bmatrix} \beta_{1,1,j} & \beta_{1,2,j} & \cdots & \beta_{1,p,j} \\ \beta_{2,1,j} & \beta_{2,2,j} & \cdots & \beta_{2,p,j} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{p,1,j} & \beta_{p,2,j} & \cdots & \beta_{p,p,j} \end{bmatrix}$$

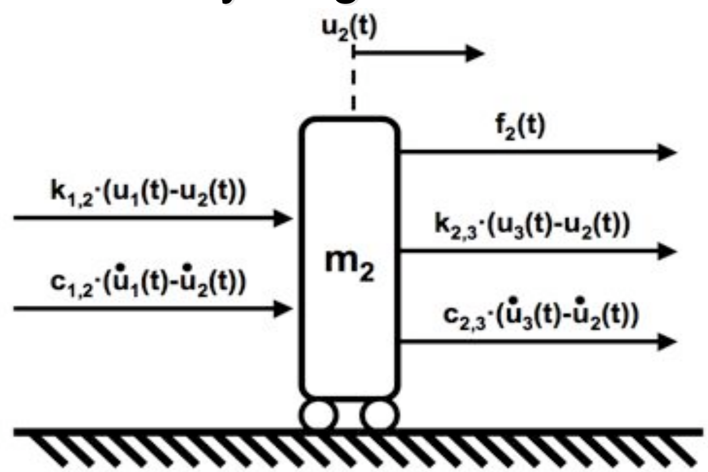
- **Communication/Data intensive:**
  - Require all auto and cross covariances up to lag q;
  - For a node to fit its rows in model:
    - $O(p^2)$ data hops are needed per time step;
- **Model prone to over-fitting:**
  - Complete AR(q) model has $(q \cdot p^2)$ parameters
  - Typical application:
    - 50 nodes, 40 lags ⇒ 10,000 parameters

"Spatial" Markov property

Diagram of vibrating system:



Free body diagram for mass i=2:



- **If state variables were observed:** $\mathbf{y}_i(t) = \begin{bmatrix} u_i(t) & \dot{u}_i(t) \end{bmatrix}^T$
  - Conditional independence structure:

$$\mathbf{y}_i(t) \mid \mathbf{y}_{\mathcal{N}_i}(t) \perp \mathbf{y}_j(t), \text{ for all } j \notin \mathcal{N}_i$$

- **We only observe accelerations:**
  - Spatial Markov property will be used as an approximation

Banded models
Definitions and advantages

$$\ddot{\mathbf{u}}(k) = \sum_{j=1}^{q} \mathbf{B}_j \ddot{\mathbf{u}}(k-j) + \varepsilon(k)$$

- **The banded AR(q) model has banded coefficient matrices:**

$$\mathbf{B}_j = \begin{bmatrix} \beta_{1,1,j} & \beta_{1,2,j} & 0 & 0 & 0 & \cdots \\ \beta_{2,1,j} & \beta_{2,2,j} & \beta_{2,3,j} & 0 & 0 & \cdots \\ 0 & \beta_{3,2,j} & \beta_{3,3,j} & \beta_{3,3,j} & 0 & \cdots \\ 0 & 0 & \beta_{4,3,j} & \beta_{4,4,j} & \beta_{4,5,j} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

(w + 1) non-zero coefficients per row

Coefficients can be determined from auto-covariance of (w+1) variables

- **Communication requirements:**
  - Each node only requires data from w neighbors:
    - Total traffic on network becomes O(p·w) data hops per epoch;

- **Number of parameters:**
  - Reduced to O(p·w·q) from $O(p^2 \cdot q)$

Estimation of parameters
Least Squares Minimization

- **Define:**

$$\ddot{\mathbb{V}} = \begin{bmatrix} \ddot{\mathbf{u}}(q+1)^T \\ \ddot{\mathbf{u}}(q+2)^T \\ \ddot{\mathbf{u}}(q+3)^T \\ \cdots \\ \ddot{\mathbf{u}}(T-1)^T \\ \ddot{\mathbf{u}}(T)^T \end{bmatrix}, \ddot{\mathbb{U}} = \begin{bmatrix} \ddot{\mathbf{u}}(q)^T & \ddot{\mathbf{u}}(q-1)^T & \cdots & \ddot{\mathbf{u}}(1)^T \\ \ddot{\mathbf{u}}(q+1)^T & \ddot{\mathbf{u}}(q)^T & \cdots & \ddot{\mathbf{u}}(2)^T \\ \ddot{\mathbf{u}}(q+2)^T & \ddot{\mathbf{u}}(q+1)^T & \cdots & \ddot{\mathbf{u}}(3)^T \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{\mathbf{u}}(T-2)^T & \ddot{\mathbf{u}}(T-3)^T & \cdots & \ddot{\mathbf{u}}(T-q-1)^T \\ \ddot{\mathbf{u}}(T-1)^T & \ddot{\mathbf{u}}(T-2)^T & \cdots & \ddot{\mathbf{u}}(T-q)^T \end{bmatrix}, \mathbb{B} = \begin{bmatrix} \beta_{1,1,1} & \beta_{2,1,1} & \cdots & \beta_{p,1,1} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,p,1} & \beta_{2,p,1} & \cdots & \beta_{p,p,1} \\ \beta_{1,1,2} & \beta_{2,1,2} & \cdots & \beta_{p,1,2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,p,2} & \beta_{2,p,2} & \cdots & \beta_{p,p,2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,1,q} & \beta_{2,1,q} & \cdots & \beta_{p,1,q} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,p,q} & \beta_{2,p,q} & \cdots & \beta_{p,p,q} \end{bmatrix}.$$

AR model prescribes:

$$\ddot{\mathbb{V}} = \ddot{\mathbb{U}}\mathbb{B} + \text{residuals}$$

Sum of squared residuals:

$$SSR = \text{tr}\left[\left(\ddot{\mathbb{V}} - \ddot{\mathbb{U}}\mathbb{B}\right)^T\left(\ddot{\mathbb{V}} - \ddot{\mathbb{U}}\mathbb{B}\right)\right] = \sum_{j=1}^{p}\left[\left(\ddot{\mathbb{V}}_{\bullet,j} - \ddot{\mathbb{U}}\mathbb{B}_{\bullet,j}\right)^T\left(\ddot{\mathbb{V}}_{\bullet,j} - \ddot{\mathbb{U}}\mathbb{B}_{\bullet,j}\right)\right]$$

Estimation of parameters
Least Squares Minimization

$$SSR = \mathrm{tr}\left[\left(\ddot{\mathbb{V}} - \ddot{\mathbb{U}}\mathbb{B}\right)^T \left(\ddot{\mathbb{V}} - \ddot{\mathbb{U}}\mathbb{B}\right)\right] = \sum_{j=1}^{p} \left[\left(\ddot{\mathbb{V}}_{\bullet,j} - \ddot{\mathbb{U}}\mathbb{B}_{\bullet,j}\right)^T \left(\ddot{\mathbb{V}}_{\bullet,j} - \ddot{\mathbb{U}}\mathbb{B}_{\bullet,j}\right)\right]$$

- **One uncoupled problem for each sensor:**
  - Seemingly unrelated regressions (SUR)
  - Solve p smaller problems:

$$\hat{\mathbb{B}}_{\bullet,j} = \arg\min_{\mathbb{B}_{\bullet,j}} \left[\left(\ddot{\mathbb{V}}_{\bullet,j} - \ddot{\mathbb{U}}\mathbb{B}_{\bullet,j}\right)^T \left(\ddot{\mathbb{V}}_{\bullet,j} - \ddot{\mathbb{U}}\mathbb{B}_{\bullet,j}\right)\right]$$

For the full model

$$\hat{\mathbb{B}}_{\bullet,j} = \left[\ddot{\mathbb{U}}^T\ddot{\mathbb{U}}\right]^{-1} \left[\ddot{\mathbb{U}}^T\ddot{\mathbb{V}}_{\bullet,j}\right]$$

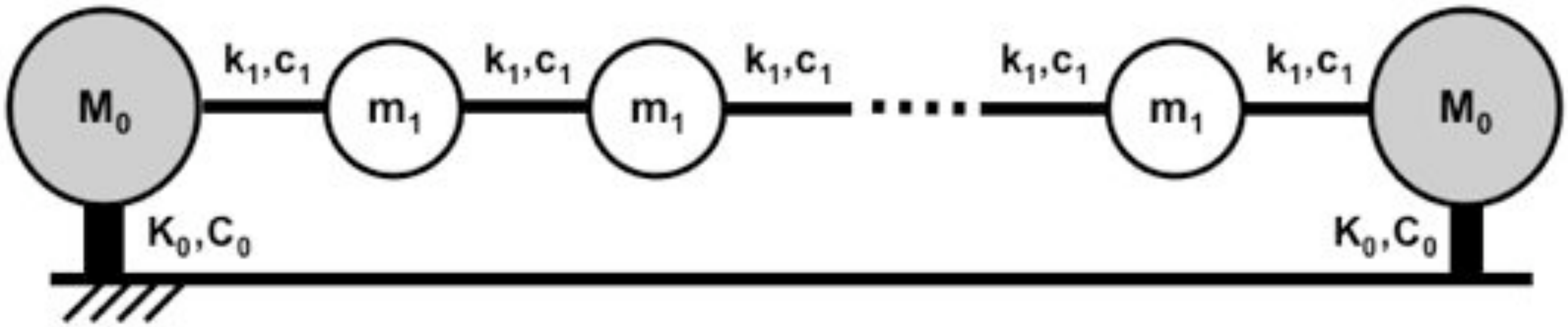
All correlations are needed

For restricted model

$$\hat{\mathbb{B}}_{\mathcal{N}_j,j} = \left[\ddot{\mathbb{U}}^T_{\mathcal{N}_j}\ddot{\mathbb{U}}_{\mathcal{N}_j}\right]^{-1} \left[\ddot{\mathbb{U}}^T_{\mathcal{N}_j}\ddot{\mathbb{V}}_{\bullet,j}\right]$$

Only local correlations are needed

The "Bridge 01" example
Modes of vibration: Mode parameters



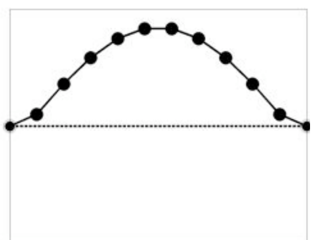
- **Modal parameters for "Bridge 01" example:**

Mode #	Freq.	Damping Ratio	Mode #	Freq.	Damping Ratio
1	1.57 Hz	5.00%	6	6.53 Hz	5.11%
2	3.00 Hz	4.00%	7	7.61 Hz	5.66%
3	4.03 Hz	4.12%	8	8.52 Hz	6.15%
4	4.67 Hz	4.31%	9	9.21 Hz	6.53%
5	5.44Hz	4.61%	10	9.63 Hz	6.77%

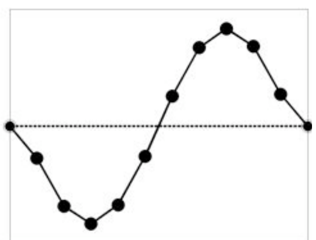
The “Bridge 01” example
Modes of vibration: Mode shapes

- **Mode shapes for “Bridge 01” example:**

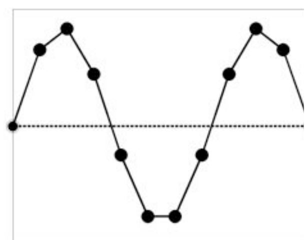
1st mode



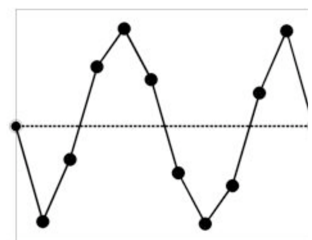
2nd mode



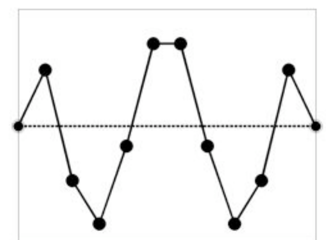
3rd mode



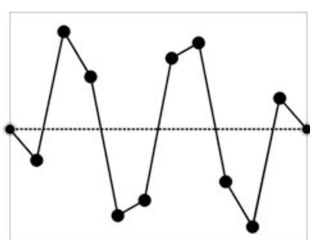
4th mode



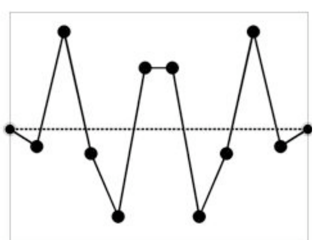
5th mode



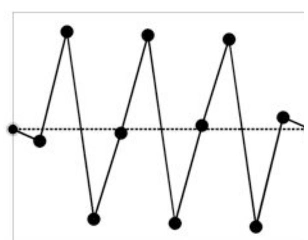
6th mode



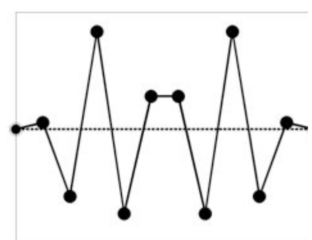
7th mode



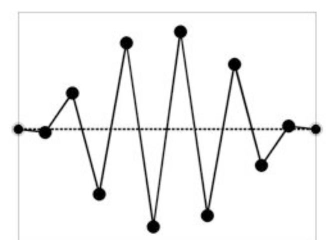
8th mode



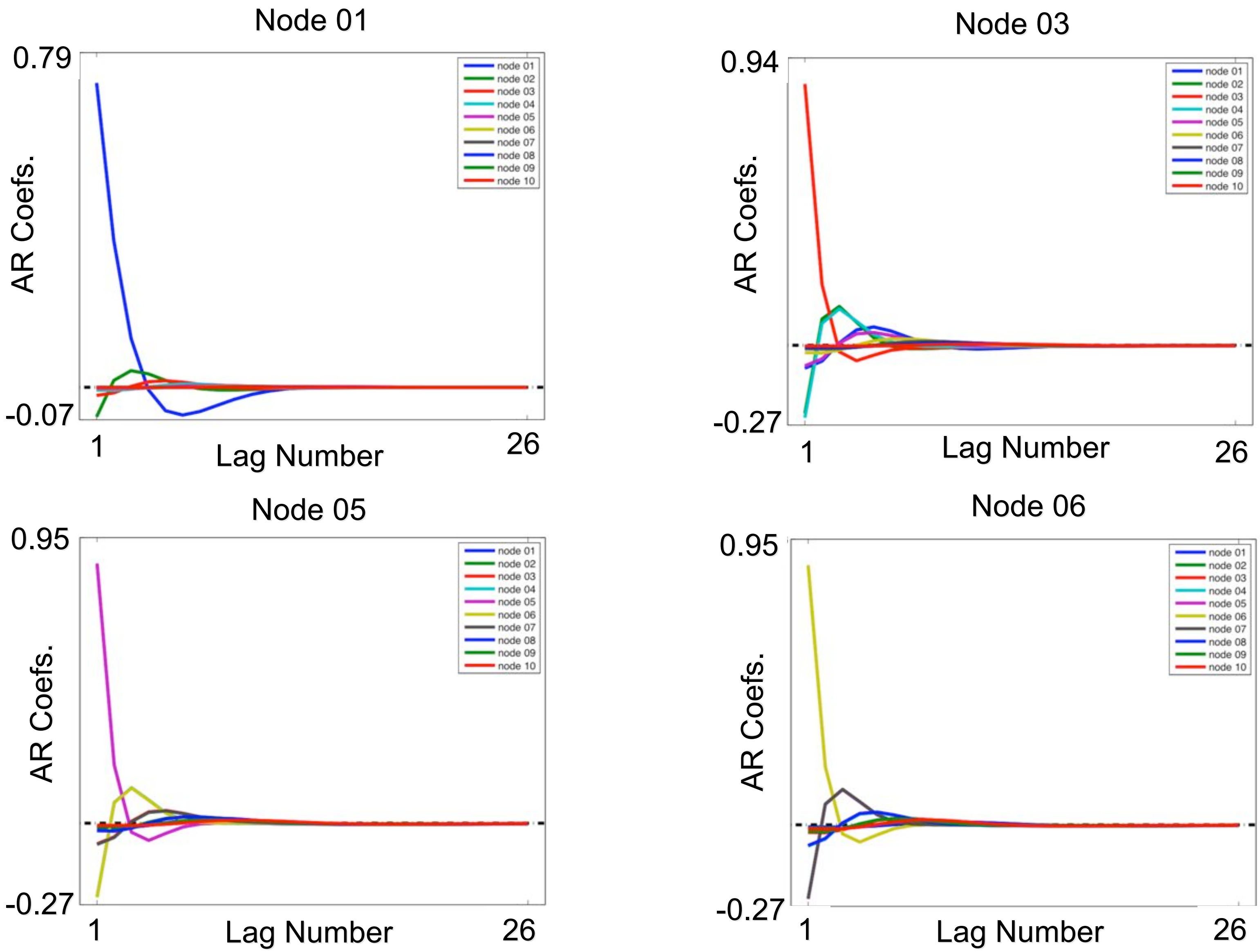
9th mode



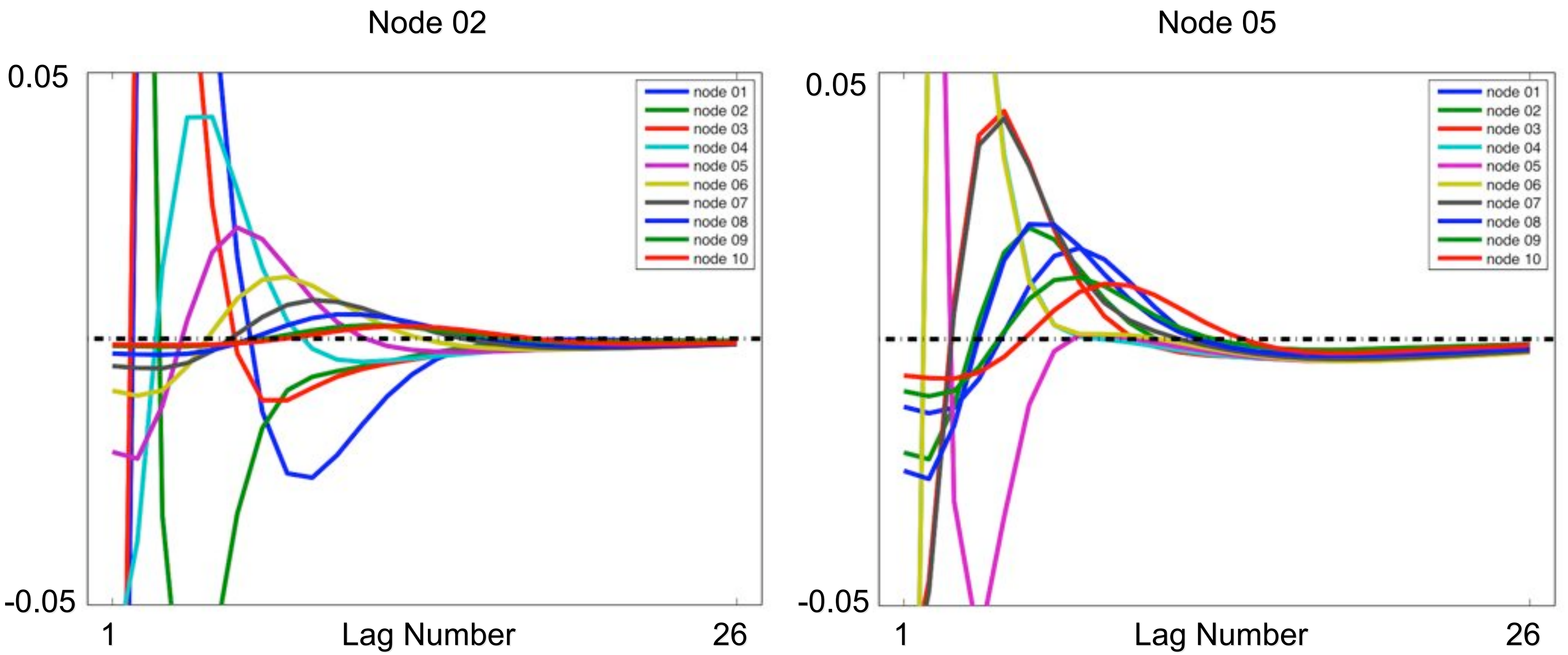
10th mode



Simulation example: population parameters AR(q) coefficients



Simulation example: Population parameters AR(q) coefficients

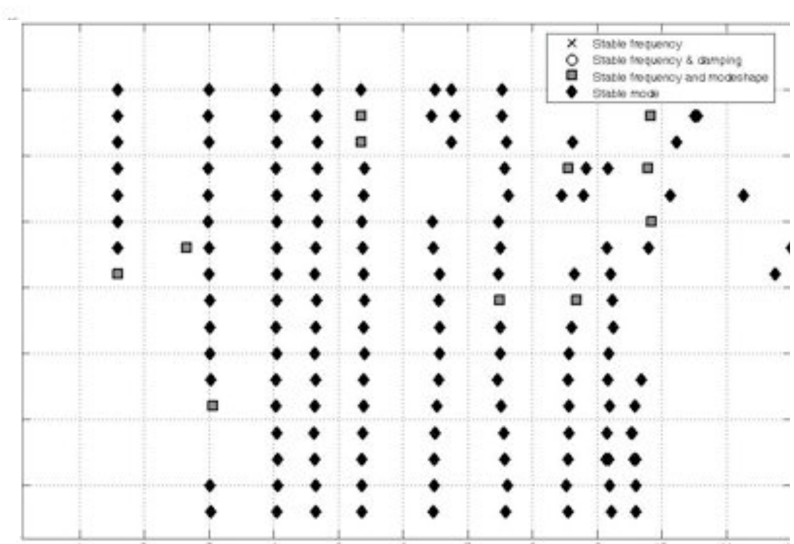


Simulation example: results
Stabilization plots

- How modes change as number of lags changes:
- 40 Hz, plots for a single sample

W=2

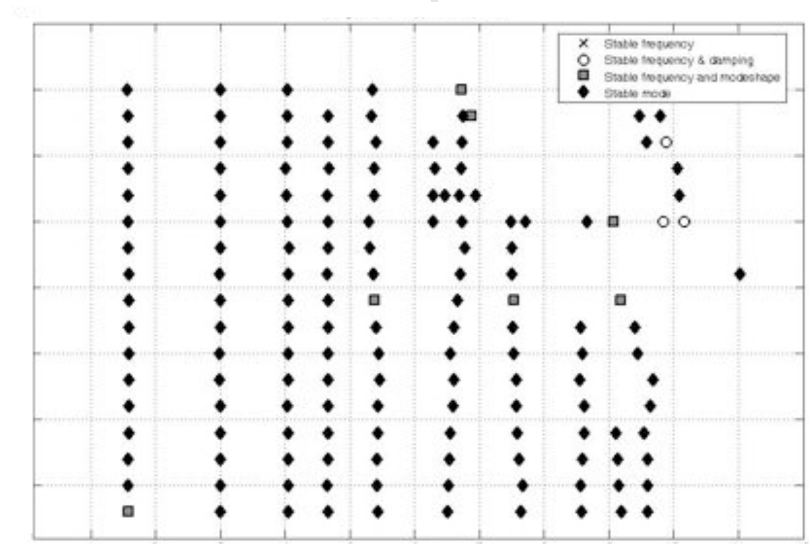
Model order



1 5 10

Frequency of mode
(Hz)

W=p



1 5 10

Frequency of mode
(Hz)

Summary

- **Application based model selection:**
  - Using bridge structure
  - Taking communication constraints into account
  - Built-in regularization

- **Future work/Extensions:**
  - Methods for choosing band width in the AR coefficient matrix
  - Analysis of spatially dense network of sensors
  - On-line fitting of coefficients (adaptive filtering/LMS)
  - Change point detection