

PubH 7475/8475 Homework 5 (Spring 2022)

Due on April 20, 2022

1. Apply 1) K-means or K-medoids or kernel K-means; 2) Normal mixture model-based clustering; 3) spectral clustering to one of the following two data sets, and use a method to select the number of clusters (and possibly other parameters too): (20-20-20=60 pts)

- NCI (NCI60) microarray data: there are $p = 6830$ predictors (i.e. genes).
- Spam data: there are $p = 57$ variables (in the Data file) to distinguish two classes, spam (coded as 1) and email (coded as 0).

For the NCI60 data, you may want to first apply some screening procedure for gene selection (**without** using the class label information), or apply some dimension reduction technique such as PCA, before clustering. For the Spam data, to save computing time, you may consider only a smaller random subset of the data (e.g. $n = 200$ to 1000). But these are optional.

You need to assess 1) the number of clusters (and other possible parameters) and 2) how well the clusters predict the classes.

Please attach your computer program and relevant output.

2. (EM for mixture model) We have iid observations (1-dim) x_1, \dots, x_n from the distribution

$$f(x; \Theta) = \sum_{k=1}^K \pi_k \phi(x; \mu_k, \sigma_k^2),$$

where Θ represents all unknown parameters, $0 \leq \pi_k \leq 1$ for and $1 \leq k \leq K$ and $\sum_{k=1}^K \pi_k = 1$, and $\phi(x; \mu_k, \sigma_k^2)$ is the density function for a Normal distribution $N(\mu_k, \sigma_k^2)$. Derive the EM algorithm to estimate Θ . (20 pts)

3. (**7000 optional; 8000 required**) Suppose we observe x_1, \dots, x_n iid from $\text{Bin}(1, p)$.
 - (a) Derive the MLE \hat{p} for p . (5 pts)
 - (b) Suppose that in addition to x_1, \dots, x_n , we have another m iid observations y_1, \dots, y_m from $\text{Bin}(1, p)$ that are randomly missing. Derive the EM algorithm for estimating p , and show whether it is the same as or better than \hat{p} . (15 pts)
4. (**8000**) Choose two papers from the lists given under Weeks 11-13 on the course Updates page: summarize the main points of and comment on each paper. (20 pts)