WinBUGS Homework

Consider the data set given on the web at
http://www.biostat.umn.edu/ sudiptob/pubh5485/WinBUGSHW1.txt.
The first column represents land value (expressed in logarithms) of 389 locations near and around the
metropolitan region in Chicago. The next three columns in the file represent distances (scaled appropriately
to lie between 0 and 1 for numerical stability) of the locations from points of interest: the first of these is
distance from Lake Michigan, the second is from Midway airport, and the third is from O’Hare airport.

It is desired to fit a simple linear regression model of the form:

\[ Y = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + \epsilon. \]

Here \( \epsilon \sim N(0, \tau^2) \), where \( \tau^2 = 1/\sigma^2 \) is the precision. Fit this model in a Bayesian setting using the following
tree prior combinations and three parallel MCMC chains with initial values given below

1. \( \beta \)'s have flat priors; \( \tau^2 \) has Gamma \( G(0.000001, 0.000001) \) prior. Initial values for chain 1: \( \beta_0 = \ldots = \beta_3 = 0; \ \tau^2 = 1.0, \) for chain 2: \( \beta_0 = \ldots = \beta_3 = 100; \ \tau^2 = 0.1 \) and for chain 3: \( \beta_0 = \ldots = \beta_3 = -100; \ \tau^2 = 10.0 \)

2. Each \( \beta \) has \( N(0, 1E - 8) \) (precision equals \( 1E - 8 \)); \( \tau^2 \sim G(0.0001, 0.0001) \). Initial values for chain 1: \( \beta_0 = \ldots = \beta_3 = 0; \ \tau^2 = 1.0, \) for chain 2: \( \beta_0 = \ldots = \beta_3 = 100; \ \tau^2 = 0.1 \) and for chain 3: \( \beta_0 = \ldots = \beta_3 = -100; \ \tau^2 = 10.0 \)

3. Each \( \beta \) has \( U(-5000, 5000); \ \sigma \sim U(0, 1000) \). Initial values for chain 1: \( \beta_0 = \ldots = \beta_3 = 0; \ \sigma = 1.0, \) for
chain 2: \( \beta_0 = \ldots = \beta_3 = 100; \ \sigma = 0.1 \) and for chain 3: \( \beta_0 = \ldots = \beta_3 = -100; \ \sigma = 10.0 \)

Present your output in terms of trace-plots for the parallel chains, reporting the burn-in you used, and
present the density plots and 95% credible intervals for the parameters in your model. Comment on the
sensitivity of the estimates to the three prior combinations.